MICROLOCAL and SPECTRAL THEORY

Conference in honour of RICHARD MELROSE

Abstracts

Nalini Anantharaman: Quantum ergodicity on graphs: from spectral to spatial delocalization

Abstract: After reviewing various notions of localization/delocalization, we will more specifically look at the notion of quantum ergodicity and we will prove (under additional assumptions) the following result : if an infinite tree possesses purely absolutely continuous spectrum, and if this tree is "approximated" in a certain sense by large finite graphs, then the eigenfunctions of the latter are more or less equidistributed. Note that this is a deterministic result; for certain classes of random graphs, "quantum unique ergodicity" has been proven by Yau, Huang, Bauerschmidt and Knowles.

Yaiza Canzani: Understanding the growth of Laplace eigenfunctions

Abstract: In this talk we will discuss a new approach to understanding eigenfunction concentration. We characterize the features that cause an eigenfunction to saturate the standard supremum bounds in terms of the distribution of L^2 mass along geodesic tubes emanating from a point. We also show that the phenomena behind extreme supremum norm growth is identical to that underlying extreme growth of eigenfunctions when averaged along submanifolds. Using the description of concentration, we obtain quantitative improvements on the known bounds in a wide variety of settings. This is joint work with Jeff Galkowski.

Semyon Dyatlov: Control of eigenfunctions on negatively curved surfaces

Abstract: Given an L^2-normalized eigenfunction with eigenvalue λ^2 on a compact Riemannian manifold (M, g) and a nonempty open set $\Omega \subset M$, what lower bound can we prove on the L^2-mass of the eigenfunction on Ω? The unique continuation principle gives a bound for any Ω which is exponentially small as $\lambda \to \infty$. On the other hand, microlocal analysis gives a λ-independent lower bound if Ω is large enough, i.e. it satisfies the geometric control condition.

This talk presents a λ-independent lower bound for any set Ω in the case when M is a negatively curved surface, or more generally a surface with Anosov geodesic flow. The proof uses microlocal analysis, the chaotic behavior of the geodesic flow, and a new ingredient from harmonic analysis called the Fractal Uncertainty Principle. Applications include control for Schrödinger equation and exponential decay of damped waves. Joint work with Jean Bourgain, Long Jin, and Stéphane Nonnenmacher.

Colin Guillarmou: Geodesic stretch, marked length spectrum and some pressure

Abstract: I will review some recent work with T. Lefeuvre and G. Knieper on the rigidity problem for the marked length spectrum of negatively curved compact manifolds, i.e. the length of closed geodesics ordered by their free homotopy classes. The problem consists in proving that the marked length spectrum determines uniquely the metric up to isometry. For that purpose, we use the notion of geodesic stretch and pressure metric that goes back to Thurston, Mc Mullen and others, as well as some microlocal tools related to Melrose radial point estimates and recent works of Faure-Sjöstrand and Dyatlov-Zworski.

Peter Hintz: Linear stability of slowly rotating Kerr black holes

Abstract: I will describe joint work with Dietrich Häfner and András Vasy in which we study the asymptotic behavior of linearized gravitational perturbations of Schwarzschild and slowly rotating Kerr black hole spacetimes. We show that solutions of the linearized Einstein equation decay at an inverse polynomial rate to a stationary solution (given by an infinitesimal variation of the mass and angular momentum of the black hole), plus a pure gauge term. Our proof uses a detailed description of the low energy resolvent of an associated wave equation on symmetric 2-tensors. The underlying estimates originate in Melrose's microlocal approach to Euclidean scattering theory.

Laure Saint-Raymond: Internal waves in a domain with topography

Abstract: Stratification of the density in an incompressible fluid is responsible for the propagation of internal waves. In domains with topography, these waves exhibit interesting properties. In particular, numerical and lab experiments show that in 2D these waves concentrate on attractors for some generic frequencies of the forcing (see Dauxois et al). At the mathematical level, this behavior can be analyzed in the inviscid case with tools from spectral theory and microlocal analysis. The weakly viscous case exhibits a slightly different phenomenology, and requires new mathematical ideas.

Peter Sarnak: Spectra of metric graphs and summation formulae

Abstract: The geometric optics trace formula gives the singular support of wave trace on a compact Riemannian manifold. In the case of of a one dimensional singular manifold, that is a metric (or quantum) graph, this formula is exact and gives a crystalline measure. We examine the additive structure of the spectra of such metric graphs and their summation formulae. A key ingredient is the diophantine theory of a torus, Skolem, Lang ... and generalization. Joint work with P. Kurasov.

Terry Tao: Embedding the Heisenberg group into a Euclidean space with bounded distortion

Abstract: From the work of Pansu and Semmes it is known that the Heisenberg group (with the Carnot-Caratheodory metric) cannot embed into Euclidean space (or even Hilbert space) in a bilipschitz fashion. However if one "snowflakes" the metric then this becomes possible thanks to work of Assouad. There is a lower bound on the distortion in doing so due to Austin, Naor, and Tessera; we show that this bound can be attained while embedding into a bounded dimensional Euclidean space, answering a question of Naor and Neiman in the negative. Our argument uses a "para-differential" iteration inspired by the Nash-Moser iteration scheme (the key difficulty being that of solving a certain differential equation without loss of derivatives in the solution) as well as some basic heat kernel estimates for the Heisenberg group.

Akshay Venkatesh: Microlocal analysis and representation theory

Abstract: If a Lie group G acts on a manifold M, one can examine how $L^2(M)$ decomposes into irreducible G-representations. I will discuss this from a microlocal viewpoint, emphasizing questions that come out of the theory of automorphic forms.