

## Math 112-40, Mr. Church, Homework 11

Due at the beginning of class on Wednesday, November 25.

Please staple your homework.

Note that this assignment has two pages.

- Is 85,836 divisible by 9? Is 123,456,789 divisible by 9? How about 987,654,321? How about 385,291,476? (If you don't know the trick to check whether a number is divisible by 9, start asking around—one of your friends/roommates/stingrays can tell you.)
  - Describe the method you used to find the answers in part (a). (You should describe your method well enough that I could follow your instructions and do it myself based on your description.)
- Exercise 6.19.
- Recall that the ring  $\mathbb{Z} \times \mathbb{Z}$  is the set of pairs of integers  $(a, b)$ , where addition and multiplication are “in each coordinate separately”:

$$(a, b) + (c, d) = (a + c, b + d) \quad \text{and} \quad (a, b) \cdot (c, d) = (a \cdot c, b \cdot d).$$

For example,

$$(2, 3) + (10, 15) = (12, 18) \quad \text{and} \quad (3, 4) \cdot (2, 0) = (6, 0).$$

The multiplicative identity in this ring is the element  $(1, 1)$ .

- Find all the elements of  $\mathbb{Z} \times \mathbb{Z}$  that have a multiplicative inverse; that is, find all the elements of  $U(\mathbb{Z} \times \mathbb{Z})$ . (We actually did this in class a long time ago.)
- What is the order of the group  $U(\mathbb{Z} \times \mathbb{Z})$ ?
- For each of the elements you found, find the order of that element in the group  $U(\mathbb{Z} \times \mathbb{Z})$ .

Now consider the similar ring  $\mathbb{R} \times \mathbb{R}$ , whose elements are pairs of real numbers  $(x, y)$  and addition and multiplication are again “in each coordinate separately”:

$$(x, y) + (z, w) = (x + z, y + w) \quad \text{and} \quad (x, y) \cdot (z, w) = (x \cdot z, y \cdot w).$$

For example,

$$(\pi, 4) + (\sqrt{2}, -5) = (\pi + \sqrt{2}, -1) \quad \text{and} \quad (\pi, 4) \cdot \left(\frac{1}{\pi}, \frac{1}{8}\right) = \left(1, \frac{1}{2}\right).$$

- For which elements  $(x, y)$  of  $\mathbb{R} \times \mathbb{R}$  will  $(x, y)$  have a multiplicative inverse? If it has one, what is the multiplicative inverse of  $(x, y)$ ?

4. Exercise 6.30.
5. Exercise 6.31.
6. Exercise 6.16.