

### Math 112-40, Mr. Church, Homework 3

Due at the beginning of class on Monday, October 12.

Please staple your homework.

**Note:** there will be no late penalty if this homework is turned in by Wednesday. However, if you turn in the homework after Monday, there is no guarantee that you will get it back before the midterm on Monday, October 19.

1. As in class, we define Even-Odd Arithmetic as the set  $S = \{E, O\}$ , with addition defined by  $E + E = E$ ,  $O + E = E + O = O$ , and  $O + O = E$ , and multiplication defined by  $E \cdot E = E$ ,  $O \cdot E = E \cdot O = E$ , and  $O \cdot O = O$ .<sup>1</sup>
  - (a) Verify for yourself that addition and multiplication are commutative and associative (there's no need to write anything). The distributive law also holds.
  - (b) What is the additive identity in this number system? Does every element have an additive inverse? How many solutions does the equation  $x = -x$  have in this number system?
  - (c) What is the multiplicative identity in this number system? Does every element have a multiplicative inverse?
  - (d) Let 1 be the multiplicative identity in Even-Odd Arithmetic. Does the equation  $x^2 = -1$  have a solution in this number system?
  - (e) Let 0 be the additive identity in Even-Odd Arithmetic. Is it true that  $0 \cdot x = 0$  for every possible value of  $x$ ?

Powers in number systems are defined using multiplication: for example,  $x^3$  means  $x \cdot x \cdot x$ .

2. Exercise 1.28.
3. Exercise 1.29.
4. Exercise 1.30. (A perfect square is a number which is the square of another number. For example,  $4^2 = 4 \cdot 4 = 6$ , so 6 is a perfect square in Ones' Digit Arithmetic.)
5. Exercise 1.31.
6. In any number system, we say that  $y$  is a square root of  $x$  if  $y \cdot y = x$ . For example, in the integers, 5 is a square root of 25;  $-5$  is also a square root of 25.
  - (a) In One's Digit Arithmetic, how many numbers have two square roots? How many have exactly one square root? Does any number have more than two square roots?
  - (b) Which numbers are their own square root?
  - (c) Does  $-1$  have a square root?

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<sup>1</sup>These are the rules for adding and multiplying even and odd numbers;  $E$  corresponds to even numbers, and  $O$  corresponds to odd numbers.