Math 112-40, Mr. Church, Homework 6

Due at the beginning of class on Wednesday, November 4.
Please staple your homework.

1. In class on Friday we proved that every number n is divisible by some prime number, but in fact there is a stronger statement:

   Prove that every number n is divisible by some prime number p satisfying $p \leq \sqrt{n}$.

2. In class on Friday we proved that if you assume there are only finitely many primes $p_1, p_2, \ldots, p_k$, you get a contradiction by building the number $N = p_1 \times p_2 \times \cdots \times p_k + 1$.

   Work through the details of the argument to show that you get a contradiction if you assume the only prime numbers are 2, 3, 5, and 7. What number N do you build? Why can't it be divisible by 2 or 3 or 5 or 7? What is its prime factorization?[1]

3. Exercise 3.16.

4. Exercise 3.17.

5. Challenge question[2]: Prove the only "triplet of primes" is 3/5/7; this fact was mentioned at the end of class on Friday. Mathematically, the question is: prove that the only number n for which n, $n+2$, and $n+4$ are all primes is $n = 3$.

[1] If you want another reference for how the proof goes, the book's proof of this theorem is on pages 101–102.

[2] Quite difficult; do the best you can.