Math 112-40, Mr. Church, Homework 9

Due at the beginning of class on Wednesday, November 18.

Please staple your homework.

For each of the following pairs of numbers, you should use the Euclidean algorithm to
a) Find the GCD of the two numbers, and
b) Write the GCD as a linear combination of the two numbers
(that is, find numbers m and n so that $\gcd(a, b) = ma + nb$).

1. 112 and 70.
2. 28 and 104.
3. 81 and 9.
4. 479 and 213.
5. 91 and 65.

Challenge problems—no points, but you get your name in the liner notes of *Bruce Willis's Math Ability*'s first album, "Bruce Willis: Divide Hard".
(Partial tracklist: The Fifth Element, The Sixth Sense, Lucky Number S7evin, The Whole Nine Yards, Ocean's Eleven, Twelve Monkeys, Sixteen Blocks.)

6. Using the following theorem,

   Theorem: every common divisor of a and b divides $\gcd(a, b)$.

   show that every common multiple of c and d is a multiple of $\operatorname{lcm}(c, d)$.

7. Using the conclusion of the previous question, show that

   $$x \equiv y \pmod{a} \qquad \text{and} \qquad x \equiv y \pmod{b}$$

   if and only if

   $$x \equiv y \pmod{\operatorname{lcm}(a, b)}.$$

   [Note that I stated this theorem incorrectly in class; this is the corrected version.]