Math 113: Linear Algebra and Matrix Theory

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Homework 2

Due Wednesday, October 7 in class.

Do all the following exercises:

2A.11			
2B.5	2B.7		
2C.1	2C.7	2C.11	2C.12
3A.11	3A.14		

Question 1. If V is a vector space over the field $\mathbf{F}$, the *dual vector space* V^* is the vector space $\mathcal{L}(V, \mathbf{F})$ of linear maps from V to $\mathbf{F}$. Explicitly, the elements of V^* are the functions $f\colon V \to \mathbf{F}$ that satisfy

$$f(v + w) = f(v) + f(w) \qquad \forall v, w \in V$$

$$f(a \cdot v) = a \cdot f(v) \qquad \forall v \in V,\ a \in \mathbf{F}$$

Assume that $\dim V = n$, and that $v_1, \ldots, v_n$ is a basis for V. Find a basis for V^*. What is $\dim V^*$?

Question 2. Let V be a vector space with basis v_1, v_2, and let W be a vector space with basis w_1, w_2, w_3. Find a basis for $\mathcal{L}(V, W)$. What is $\dim \mathcal{L}(V, W)$?

Question 3. Recall that $\mathbb{R}^\infty$ is the vector space whose elements are infinite sequences of real numbers $v = (v_1, v_2, \ldots)$, where each v_i is a real number $v_i \in \mathbb{R}$.

Let U be the subset of $\mathbb{R}^\infty$ consisting of all sequences that satisfy

$$v_i + v_{i+2} = v_{i+1} \qquad \text{for all } i.$$

a) Prove that U is a subspace of $\mathbb{R}^\infty$.

b) Let $x, y \in U$ be the elements

$$x = (0, 1, 1, 0, -1, -1, 0, 1, 1, \ldots)$$

$$y = (1, 0, -1, -1, 0, 1, 1, 0, -1, \ldots)$$

Prove that the list x, y is linearly independent.

c) Prove that x, y is a basis for U.

d) Let W be the subspace of $\mathbb{R}^\infty$ consisting of all sequences with $v_1 = 0$ and $v_2 = 0$. (You do not have to prove that W is a subspace.) Prove that $\mathbb{R}^\infty = U \oplus W$.