Math 113: Linear Algebra and Matrix Theory

Thomas Church (tfchurch@stanford.edu)
http://math.stanford.edu/~church/teaching/113-F15

Homework 7

Due Wednesday, November 11 in class.

Do all the following exercises and questions.

6C.4	**6C.6**	**6C.11**
7A.1	**7A.2**	**7A.4**

Question 1. Suppose $(e_1, \ldots, e_m)$ is an orthonormal list of vectors in V. Let $v \in V$. Prove that

$$\|v\|^2 = |\langle v, e_1\rangle|^2 + \cdots + |\langle v, e_m\rangle|^2$$

if and only if $v \in \operatorname{span}(e_1, \ldots, e_m)$.

Question 2. Let V be the vector space of infinite sequences of real numbers:

$$V = \{\, (a_1, a_2, a_3, \ldots) \mid a_i \in \mathbb{R} \,\}$$

This is an infinite-dimensional vector space over $\mathbb{R}$. Consider the forwards shift on V: let $T \in \mathcal{L}(V)$ be the operator defined by

$$T(a_1, a_2, a_3, \ldots) = (0, a_1, a_2, \ldots).$$

(a) The operator $T + I$ is given by

$$(T + I)(a_1, a_2, a_3, a_4, \ldots) = (a_1, a_1 + a_2, a_2 + a_3, a_3 + a_4, \ldots).$$

Find an inverse $(T + I)^{-1}$ for this operator.

(b) For which values of $\lambda \in \mathbb{R}$ is the operator $T - \lambda I$ non-invertible? Try to prove your answer is correct; if you cannot prove it completely, give as much justification as you can.

(c) What are the eigenvalues of T?

(d) Explain the discrepancy between your answers to (b) and (c).