Math 113: Linear Algebra and Matrix Theory

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Homework 9

(the last HW assignment)

Due Wednesday, December 2 in class

Do all the following questions.

Question 1. Let $V = \mathbb{R}^2$, and let $T \in \mathcal{L}(V)$ be an operator on V. Assume that $v \in V$ and $w \in V$ are two nonzero vectors satisfying

$$T(v) = 2v \qquad \text{and} \qquad T(w) = -w.$$

Compute the determinant $\det(T^4 + T)$.

Question 2. On HW 5, you found the minimal polynomial of the operator $T \in \mathcal{L}(\mathbb{R}^4)$ with matrix

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 1 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

Find the characteristic polynomial of T.

Question 3. Let V be an n-dimensional vector space, and let $T \in \mathcal{L}(V)$ be an operator on V. Let $\chi_T(x)$ be the characteristic polynomial of T. Which of the following implications is true?

I. If $\chi_T(x)$ has n distinct roots, then T is diagonalizable.

II. If T is diagonalizable, then $\chi_T(x)$ has n distinct roots.

III. Both I and II are true.

IV. Neither I nor II is true.

Prove that your answer is correct, by either proving or giving a counterexample for I, and either proving or giving a counterexample for II.

Question 4. Let V be a finite-dimensional complex inner product space, and let $T\colon V \to V$ be an operator on V. Prove that if T is an *isometry*, then $|\det T| = 1$.

The term "isometry" was defined in class, as part of our big table of types of operators and their properties (normal; self-adjoint; positive; isometry; orthogonal projection). If you need a reminder, they are addressed in Chapter 7C (which we did not cover in detail).

Question 5. Let V be a finite-dimensional complex inner product space, and let $T\colon V \to V$ be an operator on V. Prove that

$$\det T^* = \overline{\det T}.$$