Math 113-40, Mr. Church, Second Midterm Review

Here are some questions to help study for the second midterm. Due to obvious technical difficulties I can't include example graphs for the questions below, but you can take pretty much any examples from the homeworks, or from the book, or just make up some graphs.

1. Given a planar graph, draw the dual graph.
2. Decide whether two graphs are isomorphic (as in Q1 on HW 3A).
3. Given a graph, find a planar graph isomorphic to it. (That is, draw the graph without edges crossing, as in Q1 on HW 3C.)
4. Given a graph, determine whether it has an Eulerian circuit. If so, find one. Does the answer depend on how the graph is drawn?
5. Given a graph, reduce it down to the graph with one vertex.
6. Given a graph, find a 4-coloring, or 5-coloring, etc.
7. What is the relation between the number of vertices/edges/faces of a planar graph and the number of vertices/edges/faces of its dual graph?
8. Prove that a graph without circuits must have at least one vertex with degree 1.
9. Given a graph, find a spanning tree.
10. Does every tree have some vertex with degree ≥ 3? If not, give an example of a tree where every vertex has degree ≤ 2.
11. Given a graph, calculate its Betti number (without finding a spanning tree).
12. Draw a frieze pattern with a translation and a horizontal reflection as symmetries, but not a rotation or a vertical reflection. Does your example have any glide-reflection as a symmetry?
13. Given a frieze pattern, identify all the kinds of symmetry it has. (Start with the examples in Exercise 15.18.)
14. The frieze pattern $\cdots$XXX XXX XXX XXX$\cdots$ has all the possible symmetries that a frieze pattern can have (type VII). Are there any other letters for which this is true (replacing X with the other letter)?
15. Exercise 14.6.
16. Draw a graph that requires at least 7 colors to color legally (recall that in a coloring two adjacent vertices are not allowed to have the same color).