Math 152-37, Mr. Church, Homework 11
Due in class on Friday, November 14.
Please staple your homework.

The second question requires material from Wednesday's lecture, but you can start on the others now.

1. A u-substitution with $u = 1 + t^2$ shows that

$$\int_{-1}^{1} t\sqrt{1+t^2}\, dt = 0.$$

   Explain why $\displaystyle\int_{-1}^{1} \sqrt{t^2 + t^4}\, dt$ is *not* equal to 0, and calculate this integral.

2. (**Corrected**) Consider the parametric curve in the plane $P(t) = \left(t^2, t^3\right)$ defined for $t \in [0, 3]$. Compute the arc length of this curve.

Question 3 is on the next page.

3. (**) This question is a brief preview of improper integrals (usually not taught until Math 153). It seeks to answer the question: what is $\displaystyle\int_0^9 \frac{1}{\sqrt{x}}\,dx$? Since $\displaystyle\frac{1}{\sqrt{x}}$ is not defined at $x=0$, we need to go beyond what we have done in class.

   (a) Calculate $\displaystyle\int_1^9 \frac{1}{\sqrt{t}}\,dt$.

   (b) Define a function $g(x)$ for $x>0$ by

   $$g(x) = \int_x^9 \frac{1}{\sqrt{t}}\,dt.$$

   Give a formula for $g(x)$ in terms of x (that is, evaluate the integral).

   (c) Does $\lim_{x\to 0^+} g(x)$ exist? If not, show that it does not exist; if it does, compute the limit.

This is how an integral like $\displaystyle\int_0^9 \frac{1}{\sqrt{x}}\,dx$ is defined: as $\displaystyle\lim_{x\to 0^+}\int_x^9 \frac{1}{\sqrt{t}}\,dt$.

   (d) Calculate $\displaystyle\int_1^9 \frac{1}{t^2}\,dt$.

   (e) Define a function $h(x)$ for $x>0$ by

   $$h(x) = \int_x^9 \frac{1}{t^2}\,dt.$$

   Give a formula for $h(x)$ in terms of x.

   (f) Does $\lim_{x\to 0^+} h(x)$ exist? If not, show that it does not exist; if it does, compute the limit. (This is how $\displaystyle\int_0^9 \frac{1}{x^2}\,dx$ is defined.)

   (g) Note that $-g(x) = \displaystyle\int_9^x \frac{1}{\sqrt{t}}\,dt$. Show that for any real number N, there is some $x>0$ such that $-g(x) \geq N$.

This implies that $\displaystyle\lim_{x\to\infty}\int_9^x \frac{1}{\sqrt{t}}\,dx$ does not exist; this limit is how we would define $\displaystyle\int_9^\infty \frac{1}{\sqrt{x}}\,dx$. (We have not defined $\lim_{x\to\infty} f(x)$, so you'll have to take my word about the relation between (g) and this limit.)