Math 196-47, Mr. Church, Homework 10

Due at the beginning of class on Friday, May 22.

Please staple your homework.

Leftovers:

1. Prove that $\ker(B)$ is contained in $\ker(AB)$; that is, show that every vector in the former subspace is also in the latter subspace.

2. (a) Prove that if A is an orthogonal matrix, then $\det A = \pm 1$.

   (b) Give an example of an orthogonal matrix with $\det A = 1$, and another example with $\det A = -1$.

3. Let $\vec{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$, and $\vec{w} = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$.

   (a) Compute the cross product $\vec{u} \times \vec{v}$.

   (b) Compute the cross product $\vec{v} \times \vec{w}$.

   (c) Compute the cross product $\vec{w} \times \vec{u}$.

   (d) Use the properties of the cross product to compute $\vec{u} \times (\vec{v} + \vec{w})$ without doing any more calculations.

This week:

4. Find all the eigenvalues of the matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$.

5. Find all the eigenvalues of the matrix $A = \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix}$.

6. Find all the eigenvalues of the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ -4 & 4 & 0 \\ -6 & 9 & -1 \end{bmatrix}$. [Hint: one of them is -1.]

7. Assume that A is a 3×3 matrix with eigenvalues 1, 2, and 7. What can you say about the eigenvalues of A^2? How about A^{-1}?