Math 196-47, Mr. Church, Homework 13
Due at the beginning of class on Wednesday, June 3.
This is the last official homework assignment.

1. Exercise 8.2.6.

2. Consider $\mathbb{R}^3$ as 3-dimensional space in the usual way, with $e_1 = (1, 0, 0)$ the unit vector in the x direction, $e_2 = (0, 1, 0)$ the unit vector in the y direction, and $e_3 = (0, 0, 1)$.

   Let R be the counter-clockwise rotation by $45°$ around the z axis. This is a linear transformation (you don't have to prove this). Find the standard matrix of R.

3. For each of the following, if the answer is yes, give an example; if the answer is no, give a justification. Recall that the identity map $\mathrm{id}_n\colon \mathbb{R}^n \to \mathbb{R}^n$ is defined by $\mathrm{id}_n(x) = x$.
   (a) Is it possible to find linear transformations $T\colon \mathbb{R}^2 \to \mathbb{R}^1$ and $S\colon \mathbb{R}^1 \to \mathbb{R}^2$ so that $S \circ T = \mathrm{id}_2$?
   (b) Is it possible to find linear transformations $T\colon \mathbb{R}^1 \to \mathbb{R}^2$ and $S\colon \mathbb{R}^2 \to \mathbb{R}^1$ so that $S \circ T = \mathrm{id}_1$?
   (c) Is it possible to find a linear transformation $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ so that T is one-to-one but not onto?
   (d) Is it possible to find a linear transformation $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ so that T is onto but not one-to-one?

4. Exericse 8.2.8.

5. [This question is not for credit; however, if you complete any part of it, I am happy to make comments and give feedback on your work. If you do this question, please turn it in by Friday, separately from your homework.] "Matrix multiplication and rank", on p. 268. You can think about part 5, but no need to write anything for it.

   You may use the fact that if S and S' are two subspaces of $\mathbb{R}^n$, and S is contained in S', then $\dim S \leq \dim S'$. (You can also think about why this is true.)