Math 196-47, Mr. Church, Homework 9

Due at the beginning of class on Wednesday, May 13.

Please staple your homework.

1. Exercise 6.2.2. (You do not need to sketch anything.)

2. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be nonzero vectors in $\mathbb{R}^n$ that are pairwise orthogonal (that is, $\langle \vec{a}, \vec{b} \rangle = 0$, $\langle \vec{b}, \vec{c} \rangle = 0$, and $\langle \vec{a}, \vec{c} \rangle = 0$). Show that $\{\vec{a}, \vec{b}, \vec{c}\}$ are linearly independent.

3. Which of the following matrices are orthogonal?

   (a)
$$\begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

   (b)
$$\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

   (c)
$$\begin{bmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{bmatrix}$$

   (d)
$$\begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{bmatrix}$$

4. Prove that an orthogonal matrix can never have 2 as one of its entries. (For example, this implies $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ cannot be orthogonal.)