CAAP Math, Mr. Church, Homework 1

Due at the beginning of class on Monday, July 7

`http://www.math.uchicago.edu/~tchurch/`

1. (a) Let $\clubsuit$ be a binary operation defined for real numbers; this means that for any real numbers x and y, we have some definition of $x \clubsuit y$.

   We say that the operation $\clubsuit$ is *commutative* if for all real numbers x and y,

   $$x \clubsuit y = y \clubsuit x.$$

   We say that the operation $\clubsuit$ is *associative* if for all real numbers x, y, and z,

   $$x \clubsuit (y \clubsuit z) = (x \clubsuit y) \clubsuit z.$$

   In the following questions, if the answer is "no", then give a counterexample: a pair of numbers that shows that the operation is not commutative, or three numbers that show that the operation is not associative.

   (b) Is addition of real numbers commutative?

   (c) Is multiplication of real numbers commutative?

   (d) Is subtraction of real numbers commutative?

   (e) Is division of nonzero real numbers commutative?

   (f) Is addition of real numbers associative?

   (g) Is multiplication of real numbers associative?

   (h) Is subtraction of real numbers associative?

   (i) Is division of nonzero real numbers associative?

   (j) Define the operation $\heartsuit$ on the nonzero integers by $x \heartsuit y = x^y$. Is $\heartsuit$ commutative? Is it associative?

   (k) Define the operation $\diamondsuit$ on the integers by $x \diamondsuit y = (x + y)xy$. Is $\diamondsuit$ commutative? Is it associative?

   (l) Is it true that $x \diamondsuit y = 0$ <u>if and only if</u> $x = 0$ or $y = 0$? If not, what is the condition on x and y given by $x \diamondsuit y = 0$? (Give an exact characterization of when this happens, not just some examples.)

2. Factor the polynomial $x^2 + 1000002x + 2000000$.

3. Try to reduce each fraction to lowest terms.
   (a) Convert the decimal $0.1\overline{5}$ to a fraction.
   (b) Convert the decimal $3.\overline{142857}$ to a fraction.
   (c) Convert the decimal $0.\overline{9}$ to a fraction.

4. What is $(-1)^{1776}$?

5. Use long division of polynomials to compute the quotient and the remainder of

$$\frac{4x^3 - 11x^2 + 3x - 6}{2x - 3}.$$

6. Starting with the curve defined by $y = x^3 - 5x^2 - 4x + 20$, we apply the transformation that shifts to the right by 1 unit. Which of the following is the equation of the resulting curve?

   A. $x^3 - 2x^2 - 11x + 12$
   B. $x^3 - 5x^2 - 4x + 11$
   C. $x^3 - 8x^2 + 9x + 18$
   D. $x^3 + 10x^2 - 4x - 5$