

CAAP Math, Mr. Church, Homework 3
Due at the beginning of class on Monday, July 14
<http://www.math.uchicago.edu/~tchurch/>

You are strongly encouraged to work together, but everything you write down **must** be in your own words. If you do not staple your homework, please write your name on every sheet of paper.

1. Read sections 1.1, 1.2, and 1.3.
2. Exercise 1.2: Write the following sets in $\{x \in X | P(x)\}$ notation:
 - (a) Countries which are part of the United Nations.
 - (b) People born in 1950.
 - (c) All vegetarians.
 - (d) Numbers whose square is written with a 5.

You may let C be the set of countries and P the set of people, if this is helpful.

3. Exercise 1.3: Let $A = \{1, 3, 5, 7, 8, 11\}$ and $B = \{4, 5, 6, 7, 8\}$. What is $A \cup B$? What is $A \cap B$?
4. Exercise 1.4: Let $A = \{x \in \mathbb{Z} | x > 8\}$ and $B = \{y \in \mathbb{Z} | y \text{ is even}\}$. What is $A \cup B$? What is $A \cap B$?
5. Exercise 1.7: Let A and B be as above [$A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4, 5\}$.] Which of the following represent a function from A into B ?

$$F = \{(1, 2), (2, 3), (3, 4), (4, 5)\}$$

$$G = \{(1, 4), (2, 4), (3, 3)\}$$

$$H = \{(2, 1), (1, 3), (2, 3), (3, 5)\}$$

$$I = \{(1, 5), (3, 3)\}$$

$$J = \{(1, 6), (2, 7), (3, 8)\}$$

$$K = \{(3, 3), (2, 4), (1, 5)\}$$

6. Exercise 1.9: For integers, $f(x) = 2$.
 - (a) Is f a function?
 - (b) What is the domain of f ?
 - (c) Is f one-to-one?
 - (d) Can you choose the range of f so that f is onto?
7. Exercise 1.11: Let $A = \{2, 4, 6\}$ and $B = \{1, 2, 3\}$. Define $f: A \rightarrow B$ by saying f sends 2 to 1, 4 to 2, and 6 to 3. Then define $g: B \rightarrow A$ by saying g sends 1 to 6, 2 to 6, and 3 to 4.

- (a) What is the range of $f \circ g$?
 - (b) What are $(f \circ g)(1)$, $(f \circ g)(2)$, and $(f \circ g)(3)$?
 - (c) What is the range of $g \circ f$?
 - (d) What are $(g \circ f)(2)$, $(g \circ f)(4)$, and $(g \circ f)(6)$?
8. Exercise 1.13: Find the additive and multiplicative inverses in \mathbb{R} of the following real numbers:
- (a) 2
 - (b) 7
 - (c) -1
 - (d) -3
 - (e) 0
 - (f) $2 + \frac{1}{2}$
 - (g) 0.01
 - (h) π
9. Exercise 1.16: For each of the following sets, tell whether the set is closed under addition and whether the set is closed under multiplication. Justify your answers either by giving an explanation of why the set is closed or an example that shows it is not closed.
- (a) The real numbers
 - (b) The natural numbers
 - (c) $\{0, \frac{1}{2}, 1, \frac{3}{2}, \dots\}$
 - (d) The odd integers $\{\dots, -3, -1, 1, 3, \dots\}$
 - (e) The irrational numbers
 - (f) The negative real numbers
 - (g) The set of real numbers between 0 and 1
 - (h) The set $\{-1, 0, 1\}$
10. Exercise 1.22: If S is a set which contains the number 1 and is closed under addition and has additive inverses, what other numbers must be in the set S ?
11. Exercise 1.23: Suppose S is a set of integers.
- (a) If S is closed under multiplication and contains the number 2, what other numbers must be in the set S .
 - (b) If S also has multiplicative inverses, what other numbers must be in the set S ?