Name:____________________________

CAAP Math C, Mr. Church, Pop Quiz

Wednesday, July 23

`http://www.math.uchicago.edu/~tchurch/`

Write your name at the top of the page. You may use this page for scratch paper, but write your answer on the line provided. You have 20 minutes to complete this quiz.

1. (10 points) For prime factorizations, use either the form $18 = 2 \cdot 3 \cdot 3$ or $18 = 2 \cdot 3^2$.

   (a) Write 72 as the product of prime numbers.

   (a) ____________

   (b) Write 24 as the product of prime numbers.

   (b) ____________

   (c) Write 15 as the product of prime numbers.

   (c) ____________

   (d) What is the greatest common factor of 72 and 24?

   (d) ____________

   (e) What is the greatest common factor of 24 and 15?

   (e) ____________

2. (10 points) Let the function $f\colon \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2$ and let the function $g\colon \mathbb{R} \to \mathbb{R}$ be defined by $g(x) = x$. Let the function $h\colon \mathbb{R} \to \mathbb{R}$ be defined by $h(x) = f(x) + g(x)$.

   (a) What is $h(3)$?

   (a) ____________

   (b) What is the range of f?

   (b) ____________

   (c) The image of f?

   (c) ____________

   (d) The image of g?

   (d) ____________

   (e) Find a real number y which is **not in the range of** h; that is, a real number y such that $h(x) \neq y$ for any x.

   (e) ____________

3. (a) (5 points) We say that a number $m \in \mathbb{Z}$ is *even* if $2|m$. Prove that if a is even and b is any integer, then ab is even.

   (b) (5 points) Prove that for any $n \in \mathbb{Z}$, the quantity $n(n+1)$—which can also be written as $n^2 + n$—is even.

4. ($*$ points) **Only if you finish early**:

   - Prove that there are infinitely many primes, *or*,
   - Work through the details of the proof that there are infinitely many primes to show that $\{3, 5, 7\}$ cannot be a complete list of all the prime numbers.