Math 145. Algebraic Geometry

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Textbook. Algebraic Curves (Fulton). Available for free (legally!) at course webpage.
Office Hours. Conrad: MWF 10–10:50am and by appointment. Baranov: TuTh noon–3pm,
Homework and exams: Homework will be assigned on Fridays and will be due *in class* (at the start of lecture) on the following Friday. *The lowest homework grade will be dropped*, and a missed homework will count as a 0. Late homeworks will not be accepted for any reason whatsoever (i.e., they count as 0).

The grade will be based on 70% for homeworks (all counting equally towards the final grade) and 30% for a take-home final exam; there will be no mid-terms. It is permissable (and even encouraged!) for students to discuss the exercises with each other.

It is *EXTREMELY IMPORTANT* to do the homework every week! This subject involves an absolutely huge number of new ideas, and constant practice is the only way to absorb them all. Also, some topics used later in the course will be developed entirely in the homework.

Algebraic geometry was classically concerned with the geometric study of solutions to polynomial equations in several variables over **C**, and in the first half of this century it was put on a firm foundation by Zariski, Weil, and others, using the (then new) methods of commutative algebra. This allowed one to work over fields other than **C**, and made it possible to exploit geometric ideas in previously 'un-geometric' subjects such as number theory. This course will be concerned with developing some basic concepts of the subject over an arbitrary algebraically closed base field.

After setting up the basic structures of the theory (algebraic varieties and maps between them), which will take a while to do, we will discuss topics such as intersection theory for plane curves, blow-ups and resolution of singularities for curves, and the Riemann-Roch theorem (which is the most important tool in the study of curves). Hopefully you will gain insight into the geometric motivation for some basic notions in commutative algebra, in addition to learning how algebraic ideas can be used to study and solve geometric problems.

In order to cover a reasonable amount of material, I will assume you have been exposed to abstract algebra at the level of Galois theory (including fields of positive characteristic). We will develop all of the commutative algebra that we need. Also, since we want to think about geometric objects, it is assumed that you have some previous exposure to the concept of manifold and abstract topological space (without metric), though none of the non-trivial theory of these concepts is required. From a purely logical point of view, we will never need any results depending on calculus (though examples in the context of manifolds are certainly a big motivation for what we will do).