

MATH 154. ALGEBRAIC NUMBER THEORY

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Course website: <http://math.stanford.edu/~conrad/154Page/>

Course assistants: Aaron Landesman (aaronlandesman@stanford.edu, 381N Sloan Hall) and Evan Warner (ebwarner@stanford.edu, 384K Sloan Hall).

Office hours: 4-5pm MWF (Conrad), TuTh 4-5pm (Warner), Tu 5:30-6:30pm and Th 2-3pm (Landesman).

Prerequisites: Math 120 and 121 (elementary group theory, notion of ideal in a commutative ring, quotient of a ring by an ideal, the idea of a module and quotient by a submodule, and finite Galois theory – including finite fields). A handout on what is needed at the outset concerning modules over a PID is posted at the course website. We begin using Galois theory seriously around halfway through the course, but it does show up in low-degree situations (e.g., quadratic extensions) early in the course.

I have to assume the level of experience with algebra that is obtained from Math 120 and 121 in order that we can focus our efforts on number theory and not have to spend a huge amount of time on algebra preliminaries.

Textbooks: *Algebraic Theory of Numbers* by Pierre Samuel. A book with lots of concrete examples (especially in its exercises), but somewhat clunky theoretical development, is Marcus' *Number Fields*.

Homework/exams: Homework is posted every Wednesday on the course website and is due on the following Wednesday *at the start of class* submitted on physical paper (either hand-written or typed). *Late homework is not accepted for any reason whatsoever*, but the lowest homework grade is dropped. (A late homework counts as a zero.) Be careful not to “waste” this option by using it up too early in the quarter!

There will be a take-home final exam, and no midterms. The final grade is weighted as follows: 75% homework, 25% final exam.

Course description: The fundamental theorem of arithmetic states that any non-zero integer is expressible as a product of primes in a manner that is unique up to order of the factors and units (i.e., signs). Algebraic number theory begins with trying to understand how this generalizes (or fails to do so) in algebraic number fields. An *algebraic number field* is a finite extension of \mathbf{Q} , and an element of such a field is called an *algebraic number*. This course covers the basic structure of such fields.

In particular, we will study algebraic integers (these play the role analogous to that of \mathbf{Z} inside of \mathbf{Q}) and we will prove two of the important *finiteness theorems*: finiteness of class groups (these measure the failure of the fundamental theorem of arithmetic for rings of algebraic integers) and finite-generatedness of unit groups (this generalizes classical results centered on Pell's equation). We will include as many examples and applications as possible, and some of these will be developed in the homework. In particular, if you skip a homework then please keep in mind the possibility that it may develop some notions which will be used later in the course.