"The relevant facts of category theory hold because of the formal interconnection between the concepts involved rather than because of their substantial content (which is none)."

E. Dubuc (a category theorist!)

Ch. 2: Read §8 of Ch. 2 up to and including Theorem 8.17. For Theorem 8.14A, the proof is given by Theorem 19.3 in Matsumura, and for Theorem 8.6A, the proof is given by Theorems 26.2, 26.3 in Matsumura for the separably generated case (for strict inequality in other cases, either read Hartshorne’s reference, or else look at the proof of Theorem 41, p. 127 of Zariski-Samuel Vol. 1). Observe also that Remark 8.9.1 has a superfluous noetherian condition; as long as $f$ is locally finite type, then $\Omega^1_{X/Y}$ is of finite presentation. Finally, instead of using the affine picture in Remark 8.9.2 to define $d$, one can proceed in a purely sheaf-theoretic manner by defining $db$ for a section $b$ of $\mathcal{O}_X(U)$ to be the section of $\Omega^1_{X/Y}(U)$ induced by $p^*_2(b) - p^*_1(b) \in I((U \times_Y U) \cap W))$ (check this really makes sense, and is consistent with the ‘affine case’ definition).

Do the following exercises from last week’s reading.

Ch 2: 7.1, 7.2, *7.3, 7.4(d,e), *7.7(a,b) (for (b), explain why the map fails to be a closed immersion in characteristic 2), *7.9, *7.12.

**Extra 1:** Let $X$ be a connected normal separated noetherian scheme, $U$ a non-empty open affine subscheme not equal to all of $X$. Show that all irreducible components of $X \setminus U$ have codimension 1 (i.e., the local rings of $X$ at their generic points are 1-dimensional). In particular, there exists an effective Weil divisor $D$ whose support is exactly $X \setminus U$. This fact is used in the proof of the (important!) theorem that any abelian variety over any field is projective. One uses the group structure and properness to prove that (when $k$ is algebraically closed) $\mathcal{L}(D)$ is ample (see Mumford’s beautiful book *Abelian Varieties*).

**Extra 2:** Work out the details (with charts) for the blow-up calculations illustrated with pictures for Example 2.3.3 in Chapter 9 of Qing Liu’s book.

**Extra 3:** Let $X$ be a locally noetherian normal scheme. Let $U$ be an open subscheme whose complement has codimension at least 2. Let $i : U \to X$ be the inclusion.

(i) For any invertible sheaf $\mathcal{L}$ on $X$, show that the canonical map $\mathcal{L} \to i_*(\mathcal{L}|U)$ is an isomorphism. In particular, conclude that $\mathcal{O}_X \to i_*(\mathcal{O}_U)$ is an isomorphism.

(ii) Assume that $X$ is locally factorial (e.g., regular). Show that for any invertible sheaf $\mathcal{L}$ on $U$, there exists an invertible sheaf $\mathcal{M}$ on $X$ and an isomorphism $\mathcal{M}|U \to \mathcal{L}$. Conclude that $i_*(\mathcal{L})$ is again an invertible sheaf, with the canonical map $i^*(i_*(\mathcal{L})) \to \mathcal{L}$ an isomorphism. Conclude that the natural ‘restriction’ map $i^*$ from $X$ to $U$ induces an isomorphism of Picard groups, with $i_*$ giving the inverse.

(iii) Give an example with $U$ regular and $X$ not regular and an invertible sheaf $\mathcal{L}$ on $U$ for which $i_*(\mathcal{L})$ is not invertible.