

MATH 248B. MODULAR CURVES

Instructor: Prof. Brian Conrad, conrad@math.stanford.edu

Office: 383CC, Sloan Hall

Office hours: During daily tea.

Prerequisites: Knowledge of elliptic curves over fields, modular forms and modular curves over \mathbf{C} , algebraic number theory, and the basic theory of schemes (e.g., Math 216).

Textbooks/references: The book “Arithmetic moduli of elliptic curves” by Katz–Mazur and the paper “Les schémas de modules des courbes elliptiques” by Deligne–Rapoport will be useful resources. The books “Néron Models” (by Bosch, Lütkebohmert, Raynaud) and “Algebraic geometry and arithmetic curves” by Q. Liu will probably be useful too.

Timing: This class will *not meet on Fridays*, but it will meet 11–12:15 on Mondays and Wednesdays. On the two exceptional weeks when Monday classes are cancelled, we may hold class on Friday instead (abbreviated due to the number theory seminar at noon).

Homework/exams: There will be no exams, but there will be weekly homework posted each Wednesday.

Course description: The aim of the course is to develop from scratch the ideas surrounding the arithmetic theory of moduli spaces via the example of modular curves. More specifically, we will discuss how the analytic and algebro-geometric theories are related over \mathbf{C} , as well as how suitable moduli problems provide natural \mathbf{Z} -structures, including a “moduli interpretation” at the cusps. Among the applications, we hope to discuss both the arithmetic theory of modular forms (q -expansions, Hecke operators) as well as the construction of Galois representations attached to modular eigenforms and the relationship with representation theory of GL_2 .