Math 249B. Homework 5

This week and next week the homework is to read about group cohomology, as a supplement to the overview in class. Next week there will be a couple of exercises using it in the context of profinite groups, but for this week just stick to the purely algebraic aspects by reading in Chapter IV of Cassels-Fröhlich's "Algebraic number theory". Try to get through §1–§5 this week.

[An alternative reference, though much lighter on proofs and so not quite as helpful when first getting into the subject, is Serre's book "Local fields". Lang's book "Algebra" also discusses a bit about group cohomology, and he has quite a few exercises on it at the end of the chapter in which it is discussed, as well as an explanation of how to relate the concrete but sometimes messy cochain approach of the Cassels-Fröhlich book with the more elegant derived functor approach. There is also the book "Cohomology of number fields", but it is way too fancy for the purposes of learning class field theory, especially if you have never seen group cohomology before.]

Note in particular the discussion of group homology in §3 as a replacement of group cohomology when we replace the left-exact functor $M^G = \mathrm{Hom}_{\mathbf{Z}[G]}(\mathbf{Z}, M)$ of G-invariants (i.e., maximal G-stable subgroup of M on which G acts trivially) with the right-exact functor $M_G = \mathbf{Z} \otimes_{\mathbf{Z}[G]} M$ of G-coinvariants (i.e., maximal G-stable quotient of M on which the G-action is trivial). If you are familiar with derived functors, note that whereas group cohomology is a right-derived functor $\mathrm{H}^\bullet(G, \cdot) = \mathrm{Ext}^\bullet_{\mathbf{Z}[G]}(\mathbf{Z}, \cdot)$, group homology is a left-derived functor $\mathrm{H}_\bullet(G, \cdot) = \mathrm{Tor}^\bullet_{\mathbf{Z}[G]}(\mathbf{Z}, \cdot)$.