Math 249B. Linear algebraic groups II

Instructor: Prof. Brian Conrad, `conrad@math.stanford.edu`
Office: 383-CC
Office hours: Departmental tea, every day.
Course assistant: None.

Prerequisites: Familiarity with the theory of schemes at the level of 216A,B and knowledge of the material from my previous course on linear algebraic groups ("Linear algebraic groups I", notes available on the course webpage): basic theory of linear algebraic groups over an arbitrary field (e.g., classical examples, Jordan decomposition, unipotentce, tori, geometric conjugacy theorems) and the classification for split connected reductive groups of rank 1 (i.e., SL_2, PGL_2, or $\mathbf{G}_m$) over a general field.

Textbooks: None, but the respective textbooks of Borel, Humphreys, and Springer all called "Linear algebraic groups" are useful references. Each has its own merits (e.g., Borel systematically handles general ground fields but with an archaic algebro-geometric foundation and few exercises or examples, Springer delves much more into root systems and the refined structure theory for the reductive case with extensive exercises and examples, and Humphreys' book has a detailed treatment of structure constants for commutation relations).

Homework/exams: There will be no homework or exams.

Course description: The aim of the course is to cover the structure theory of connected reductive groups over an arbitrary field k, including (relative) root systems, rational conjugacy theorems, and the Galois-action on Dynkin diagrams, illustrated with a variety of examples (both split and non-split). Some results need to be established over algebraically closed fields early in the course as a prelude to refinements over general fields. For $k = \mathbf{R}$ and $k = \mathbf{C}$ this illuminates the theory of semisimple Lie groups, for finite k it clarifies the structure of finite groups of Lie type, and for k a local or global field it leads to vast generalizations of classical results on the arithmetic of quadratic forms and central simple algebras over such fields.