Math 249B. Alterations

Instructor: Prof. Brian Conrad, `conrad@math.stanford.edu`
Office: 383CC, Sloan Hall
Office hours: MWF, tea time (3:30–4pm) and 4–4:30pm

Prerequisites: Knowledge of schemes (including awareness about smooth and étale morphisms) and commutative algebra at the level of Matsumura's book "Commutative Ring Theory".

Textbooks: There is no required text, but it is strongly recommended that everyone in the audience get a print-out of deJong's IHES paper on alterations because this paper is the template for the course.

Homework/exams: There will be no exams or homework, but students are strongly encouraged to fill in details omitted in lecture.

Course description: In 1994, Johan de Jong astonished the algebraic geometry world by proving a weakened form of resolution of singularities (using so-called "alterations" in the role of blow-ups) that works in arbitrary characteristic and is sufficient for most applications (even in characteristic 0). Moreover, his method is so conceptual that it also applies over discrete valuation rings, giving a "semi-stable reduction theorem" for higher-dimensional varieties. These results immediately had many wonderful applications (such as Serre's positivity conjecture in commutative algebra, and the potentially semistable property for Galois representations arising from the p-adic étale cohomology of varieties over p-adic fields).

In addition to the pretty geometric ideas introduced by deJong, his proof uses many important general concepts in algebraic geometry, such as the theory of stable reduction for curves, the étale topology, Artin approximation, algebraic stacks/spaces, Raynaud–Gruson flattening, resolution of singularities for abstract surfaces, and so on. So this topic is an ideal vehicle for seeing many important methods and results being used. In this course we will work through the proofs of deJong's main theorems over fields and along the way explain as much of the background as we have time to discuss.