Math 249C. Linear algebraic groups III

Instructor: Brian Conrad, 383CC Sloan Hall, `conrad@math.stanford.edu`
Time/location: 11–11:50am MWF, 380F, Sloan Hall
Office hours: MW, 1:15–2:15pm in office and 3:30–4pm at tea
Course assistant: None.

Prerequisites: Familiarity with the theory of schemes at the level of 216 A,B and knowledge of the material from my previous two courses on linear algebraic groups: general theory over a field (including conjugacy theorems over an algebraically closed field) and the structure of split connected reductive groups via root data (Existence and Isomorphism Theorems) over an arbitrary field. Knowledge of algebraic number theory will probably be needed in later parts of the course.

Textbooks: None.

Homework/exams: There will be no homework or exams.

Course description: The aim of the course is to cover the Borel–Tits structure theory of isotropic connected semisimple groups over an arbitrary field k, going beyond the k-split case that was treated in the previous course. After a quick review of highlights from the split case, we will take up the rational conjugacy theorems (for maximal k-split tori and minimal parabolic k-subgroups), relative root systems, and the $*$-action on Dynkin diagrams.

In whatever time remains after developing the structure theory, we will discuss the cohomological classification of connected reductive (especially semisimple) groups over general fields and features specific to arithmetically interesting fields (such as for Galois cohomology, finiteness theorems, and/or reduction theory, depending on time). For $k = \mathbf{R}$ this illuminates the theory of semisimple Lie groups, for finite k it clarifies the structure of finite groups of Lie type, and for k a local or global field it leads to vast generalizations of classical results on the arithmetic of quadratic forms over such fields.