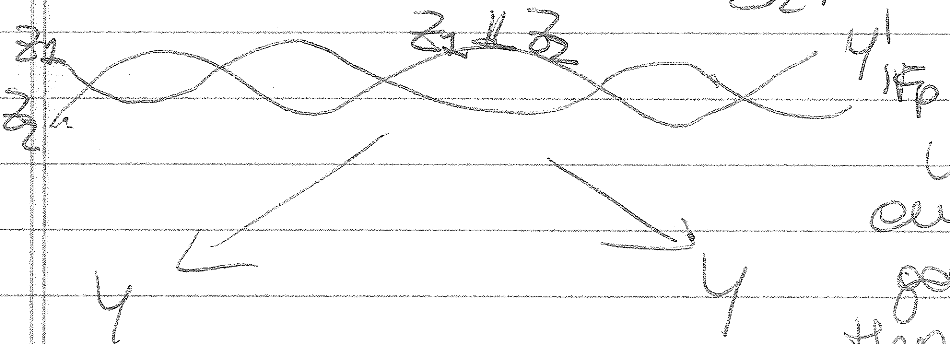
This makes sense in char p, and as everything behaves well under base change this computes the trace we needed (up to a factor of 2!)

CONGRUENCE RELATION On $\tilde{H}^1(Y_{\mathbb{F}_p}, \mathcal{O}_e)$, we have

$T_p^{alg} = F + pF^{-1}$ and pF^{-1} is the linear dual of F (up to Tate's twist) by the above.

Hence $2\,tr(T_p|_{S_2}) = tr(T_p^{alg}) = tr(F) + tr(F_{dual}) =$
$= 2\,Tr(F) \Rightarrow tr(T_p|_{S_2}) = tr(F)$.

Trick



Under each map one component Z_i goes isomorphically, the other up to Frob.

Under some hypothesis on the singularity traces and cohomology can be computed by pullback + pushforward.