1. Show that the map \( f(z) = -\frac{1}{2}(z + \frac{1}{2}) \) is a conformal equivalence of the half-disc \( D_+ := \{ z \in \mathbb{C}; \text{Im}z > 0, |z| < 1 \} \) onto \( \mathbb{H} \).

2. Show that if a holomorphic map \( f : \mathbb{D} \rightarrow \mathbb{D} \) (not necessarily an automorphism) has two distinct fixed points \( z_1 \neq z_2 \in \mathbb{D} \) (i.e. \( f(z_1) = z_1, f(z_2) = z_2 \)) then \( f(z) = z \) for all \( z \in \mathbb{D} \).

3. Prove that all biholomorphisms \( f : \mathbb{H} \rightarrow \mathbb{D} \) have the form

\[
f(z) = e^{i\theta} \frac{z - \beta}{z - \overline{\beta}} \quad \text{for some} \; \theta \in \mathbb{R}, \beta \in \mathbb{H}.
\]

4. Find \( r \in (0, 1) \) such that there exists a conformal equivalence \( \mathbb{D} \setminus \{|z - \frac{1}{2}| < \frac{1}{4}\} \) onto \( \mathbb{D} \setminus \{|z| < r\} \).

Each problem are is 10 points.