Math 116: More practice problems for the Midterm

1. Compute the integral

\[ \int_0^\pi \tan(x + ia) \, dx, \quad a \in \mathbb{C}. \]

2. Compute the integral

\[ \int_0^\infty \frac{dx}{1 + x^6}. \]

*Hint:* Integrate over the boundary of the domain \( U = \{|z| < R, 0 < \arg z < \frac{2\pi}{3}\}. \)

3. Find a holomorphic function \( f(z) \) on the domain \( U = \mathbb{C} \setminus \{ \Im z = 0, \Re z \leq 0 \} \) such the \( \arg f(z) = \phi + r \sin \phi \) for \( z = re^{i\phi}, -\pi < \phi < \pi \).

4. Find the numbers of zeroes (counted with multiplicities) of the function

\[ f(z) = e^z - 4z^n + 1 \]

in the unit disc \( \mathbb{D} \).

5. Compute the integral

\[ \frac{1}{2\pi i} \oint_{|z-a|<1} \frac{ze^z \, dz}{(z-a)^3}. \]

6. Prove that the function \( f(z) = \cos z \) define a biholomorphism

\[ f : U := \{0 < \Re z < \pi\} \to V \subset \mathbb{C} \]

and find the domain \( V = f(U) \).