RESEARCH STATEMENT

IRVING DAI

1. Introduction and Overview

My research lies in the general area of low-dimensional topology. In particular, I am interested in the development and application of *Floer homology* to geometric questions regarding knots, surfaces and manifolds in three and four dimensions. Originating out of advances in Yang-Mills theory and symplectic geometry, Floer homology provides a powerful set of invariants for use in understanding these objects and studying related problems. In my own research, I apply Floer-theoretic techniques to investigate questions involving *cobordism* and *knot concordance*.

A common construction in low-dimensional topology is to quotient out a set of topological objects by a restricted form of bordism in order to obtain a group. For example, taking the set of integer homology 3-spheres and quotienting out by the relation of homology cobordism[1] gives the *homology cobordism group* $\Theta^3_{\mathbb{Z}}$. Similarly, the *knot concordance group* $\mathcal{C}$ is defined by taking the set of knots in S^3 and declaring two knots to be concordant if they cobound a smoothly embedded annulus in $S^3 \times I$. In each case, the operation of connected sum gives the set in question a group structure, the study of which forms an active area of research.

In addition to being of intrinsic interest, questions related to these groups can help capture the divide between smooth and continuous topology in low dimensions. For example, an important theorem of Freedman states that the *topological* homology cobordism group is trivial; that is, every homology sphere bounds a contractible topological manifold (which may or may not be smoothable). This fact is related to several other fundamental results, such as Freedman's celebrated classification of simply-connected topological 4-manifolds. Thus, the complexity of $\Theta^3_{\mathbb{Z}}$ reflects the divide between between smooth and continuous topology in dimension four. Similarly, much work has gone into understanding the difference between smooth versus topological concordance, as well as related notions such as topological sliceness and topological isotopy.

In general, understanding the structure of $\Theta^3_{\mathbb{Z}}$ or $\mathcal{C}$ is very difficult, and techniques for doing so have relied on a surprisingly wide array of tools. Examples of this include work of Furuta [Fur90] and Fintushel and Stern [FS90], who showed that $\Theta^3_{\mathbb{Z}}$ has a $\mathbb{Z}^\infty$-subgroup using ideas from Yang-Mills theory; and work of Manolescu [Man16], who constrained 2-torsion in $\Theta^3_{\mathbb{Z}}$ by using Seiberg-Witten Floer homology. (Note that we still do not know whether there is any torsion in $\Theta^3_{\mathbb{Z}}$!)

- In joint work with Hom, Stoffregen, and Truong [DHST18], I showed that $\Theta^3_{\mathbb{Z}}$ admits a $\mathbb{Z}^\infty$-summand by constructing an epimorphism $\Theta^3_{\mathbb{Z}} \to \mathbb{Z}^\infty$. This was previously an open question; our proof utilized involutive Heegaard Floer homology [HM17], an invariant with roots in symplectic geometry.
- In [DHST21a], we carried out a similar construction in the context of the concordance group, using knot Floer homology (see [Ras03], [OS04a]). This gave a re-proof that the subgroup of topologically slice knots admits a $\mathbb{Z}^\infty$-summand (see [Hom15], [OSS17]).

In Section 2, I describe this and further work aimed at establishing a general program for extracting cobordism/concordance homomorphisms from the Floer homology package. Many questions regarding these techniques remain; I am hopeful that they can be applied in other settings.

A more recent idea is to employ Floer homology in the presence of a geometric symmetry: for example, an equivariant knot, or a manifold equipped with an involution. In Sections 3 and 4, I describe research focused on developing and using these techniques to study equivariant homology cobordism and concordance. Surprisingly, many results and examples obtained in this manner have connections to non-equivariant areas of low-dimensional topology.

[1] We say that Y_1 and Y_2 are homology cobordant if they cobound some smooth W such that the inclusion of each Y_i into W induces and isomorphism on homology.

- In joint work with Hedden and Mallick [DHM20], I introduced a suite of Floer-theoretic invariants aimed at detecting *corks*. These are fundamental objects in the study of smooth 4-manifold topology [Akb91]. We constructed a related equivariant homology cobordism group $\Theta^\tau_\mathbb{Z}$ and used our invariants to show that $\Theta^\tau_\mathbb{Z}$ admits a $\mathbb{Z}^\infty$-subgroup spanned by corks.
- In forthcoming work with Mallick and Stoffregen [DMS21], I use knot Floer homology to provide equivariant slice genus bounds. We use this to produce a family of strongly invertible slice knots with arbitrarily large equivariant slice genus, answering a question of Boyle-Issa [BI21]. We also show that our invariants can be used to help detect exotic pairs of slice disks, recovering some a recent result by Hayden [Hay21].

I am also interested in connections between Floer homology and other topological invariants. Chief among these is lattice homology, a combinatorial invariant for plumbed manifolds due to Ozsváth and Szabó [OS03]. This is known to be isomorphic to Heegaard Floer homology (see [OS04c], [OS04b]) in a wide range of cases.

- In joint work with Alfieri, Baldwin, and Sivek [ABDS20], I used lattice homology to compute the *framed instanton Floer homology* of Kronheimer and Mrowka [KM11] for Seifert fibered spaces. As a byproduct, we obtained an isomorphism between instanton and Heegaard Floer homology for such manifolds. This is important in light of a general conjecture that these theories coincide in all cases [KM10].
- I have used lattice homology to help compute and understand various equivariant refinements of different Floer homologies (see Section 5). These computations have formed the basis for further work involving the structure of $\Theta^3_\mathbb{Z}$.

2. Cobordism and Concordance Homomorphisms

Broad Goals. Understand the structure of various cobordism and concordance groups. Extend techniques for extracting cobordism and concordance homomorphisms from Heegaard Floer theory to other Floer homologies and/or Khovanov homology.

In a joint series of papers with Hom, Stoffregen, and Truong, I developed a general program for constructing families of $\mathbb{Z}$-valued homomorphisms for use in the study of homology cobordism and knot concordance. Using the involutive Heegaard Floer homology package of Hendricks and Manolescu [HM17], we proved:

Theorem 2.1. [DHST18, Theorem 1.1, Section 7] *There exists an infinite family of surjective, linearly independent homomorphisms*

$$\phi_i \colon \Theta^3_\mathbb{Z} \to \mathbb{Z}.$$

In particular, $\Theta^3_\mathbb{Z}$ admits a $\mathbb{Z}^\infty$-summand.

Prior to our work, the existence of a $\mathbb{Z}^\infty$-summand was an open question. It was previously known that $\Theta^3_\mathbb{Z}$ admits a $\mathbb{Z}^\infty$-subgroup (see [Fur90], [FS90]) and a $\mathbb{Z}$-summand [Frø02].

Using knot Floer homology, we constructed a similar family of homomorphisms in the context of knot concordance:

Theorem 2.2. [DHST21a, Theorem 1.1, Theorem 1.12] *There exists an infinite family of surjective, linearly independent homomorphisms*

$$\varphi_i \colon \mathcal{C} \to \mathbb{Z}.$$

These can be used to establish the existence of a $\mathbb{Z}^\infty$-summand of $\mathcal{C}_{TS}$, where $\mathcal{C}_{TS}$ is the subgroup of $\mathcal{C}$ generated by topologically slice knots.

The result regarding $\mathcal{C}_{TS}$ was previously shown by Ozsváth, Stipsicz, and Szabó using the Υ-invariant [OSS17], which can similarly be used to construct an infinite family of linearly independent homomorphisms. (See also the work of Hom [Hom15], on which [DHST21a] is based.) However, in general the homomorphisms φ_i and Υ contain different information.

In [DHST21b], we extend Theorem 2.2 to the *homology concordance group* $\widehat{\mathcal{C}}_{\mathbb{Z}}$. We use our homomorphisms to study the quotient of the homology concordance group by the subgroup $\mathcal{C}_{\mathbb{Z}}$ of knots coming from S^3.

Theorem 2.3. [DHST21b] *There exists an infinite family of linearly independent homomorphisms*

$$\varphi_{i,j} \colon \widehat{\mathcal{C}}_{\mathbb{Z}} \to \mathbb{Z}.$$

On classes in $\mathcal{C}_{\mathbb{Z}} \subset \widehat{\mathcal{C}}_{\mathbb{Z}}$, the $\varphi_{i,j}$ vanish for all $j \neq 0$, and for knots in S^3, the $\varphi_{i,0}$ coincide with the homomorphisms of Theorem 2.2. The former can be used to show that $\widehat{\mathcal{C}}_{\mathbb{Z}}/\mathcal{C}_{\mathbb{Z}}$ admits a $\mathbb{Z}^\infty$-summand.

The group $\widehat{\mathcal{C}}_{\mathbb{Z}}/\mathcal{C}_{\mathbb{Z}}$ was first studied by Hom, Levine, and Lidman [HLL18], who showed that it is infinitely generated and contains a $\mathbb{Z}$-subgroup. This was extended by Zhou in [Zho20], who obtained a $\mathbb{Z}^\infty$-subgroup.

Our work in [DHST21b] is aimed at forming a general program for extending the algebraic techniques of [DHST18] and [DHST21a] to a broader context. In order to explain this, we briefly explain the formalism of the *local equivalence group* $\mathfrak{I}$. This is obtained by quotienting the set of all possible Floer complexes by *local equivalence*, an equivalence relation modeled on the output of the Floer homology TQFT in the presence of a homology cobordism or concordance. We then obtain a homomorphism from the topological object of study (i.e., the homology cobordism or knot concordance group) into $\mathfrak{I}$. An explicit combinatorial understanding of $\mathfrak{I}$ allows us to construct further homomorphisms out of the desired topological object, factoring through $\mathfrak{I}$.

A crucial point is that $\mathfrak{I}$ varies greatly depending on the Floer-theoretic setting under consideration. For example, in the case of involutive Heegaard Floer homology, each chain complex comes equipped with a homotopy involution, which the relation of local equivalence must take into account. We may also perform algebraic operations affecting the coefficient ring, such as passing to a quotient ring or base-changing to a different coefficient ring. In [DHST18] and [DHST21a], my coauthors and I gave an explicit description of specific local equivalence groups associated to involutive Heegaard Floer and knot Floer homology, respectively. In [DHST21b], we extend this to a more general setting by defining the notion of a *grid ring*. For complexes over such rings, we give an explicit combinatorial description of the local equivalence group $\mathfrak{I}$ and describe a general method for extracting $\mathbb{Z}$-valued homomorphisms.

Interestingly, there are indications that different choices of $\mathfrak{I}$ may have direct topological meaning. For example, the ring $\mathbb{F}[U,V]/(UV)$ used in [DHST21a] is also the natural setting for the correspondence between the immersed curves picture of Hanselman, Rasmussen, and Watson [HRW16] and knot Floer homology. In fact, in this case the standard complex formalism of [DHST21a] has a natural interpretation on the immersed curves side, and the homomorphisms of Theorem 2.2 can be understood geometrically in the immersed curves picture. Here are some natural questions that I am interested in which connect the results of [DHST18], [DHST21a], and [DHST21b] to other areas of topology:

Question 2.4. What is the connection between the more extended analysis of [DHST21b] and the immersed curves picture? What is the analogue of generalizing the coefficient ring $\mathbb{F}[U,V]/(UV)$ in this context? Is there a shorter proof of Theorem 2.1, 2.2, or 2.3 on the immersed curves side?

Question 2.5. Can we apply a similar analysis in other settings, such as instanton Floer homology or Khovanov homology? For the latter, it is necessary to use versions of Khovanov homology over a ring such as $\mathbb{F}[h]$ or $\mathbb{F}[h,t]$; computational resources for these are lacking in the literature. So far, we have some preliminary results in the equivariant setting, as discussed below.

3. Strong Corks

Broad Goals. Use Floer homology to study automorphisms and symmetries of 3-manifolds. Develop Floer-theoretic invariants to help detect and construct corks.

In joint work with Hedden and Mallick, I used techniques coming from involutive Heegaard Floer homology and local equivalence to study the set of *strong corks.* Introduced by Lin, Ruberman, and Saveliev [LRS18], these are a generalization of the usual notion of a cork in which the boundary involution $\tau\colon Y \to Y$ is required to be *strongly non-extendable.* This means that τ does not extend as a diffeomorphism over *any* homology ball which Y bounds, rather than a specific contractible manifold. In [DHM20] we defined the following invariants aimed at detecting (strong) corks:

Theorem 3.1. [DHM20, Theorem 1.1] *Let Y be an integer homology sphere with involution $\tau\colon Y \to Y$. Then there are two Floer-theoretic invariants*

$$h_\tau(Y) \text{ and } h_{\iota\circ\tau}(Y)$$

associated to the pair (Y, τ). If either of these are nonzero, then τ does not extend to a self-differeomorphism of any homology ball bounded by Y.

The invariants $h_\tau(Y)$ and $h_{\iota\circ\tau}(Y)$ are valued in the same local equivalence group $\mathfrak{I}$ considered by Hendricks, Manolescu, and Zemke in [HMZ18], and are obtained by replacing the involutive Heegaard Floer map ι with τ and $\iota\circ\tau$, respectively. We furthermore defined an *involutive homology bordism group* $\Theta^\tau_{\mathbb{Z}}$; this is a refinement of the usual homology cobordism group which takes into account an involution on each end. We proved:

Theorem 3.2. [DHM20, Theorem 1.2, Theorem 1.3] *The invariants h_τ and $h_{\iota\circ\tau}$ constitute homomorphisms*

$$h_\tau, h_{\iota\circ\tau}\colon \Theta^\tau_{\mathbb{Z}} \to \mathfrak{I}.$$

These can be used to show that $\Theta^\tau_{\mathbb{Z}}$ admits a $\mathbb{Z}^\infty$-subgroup generated by strong corks.

In addition to quantifying the profusion of (strong) corks provided by our invariants, $\Theta^\tau_{\mathbb{Z}}$ is also natural to study from the viewpoint of bordism theory. Indeed, in [Mel79], Melvin showed the bordism group of diffeomorphisms Δ_3 in three dimensions is trivial.[2] The group $\Theta^\tau_{\mathbb{Z}}$ may thus be viewed as a refinement of the trivial group Δ_3, in the same way that $\Theta^3_{\mathbb{Z}}$ is a refinement of Ω_3.

Our approach in [DHM20] is quite different from previous strategies for studying corks in the literature. There, the contractible manifold W (with $\partial W = Y$) is naturally the focal point of analysis; often, W is required to be Stein so that tools such as the adjunction inequality can be employed. In contrast, in our formalism the role of W is almost entirely absent: the non-extendability of τ is inherent to the boundary Y, rather than a property of W.

Using our invariants, we were able to construct several new families of corks which were previously unknown in the literature (strong or otherwise). These included many examples demonstrating a close connection between the theory of corks and surgeries on equivariant slice knots. We were also able to use our invariants to prove that many families of existing corks in the literature are strong; in particular, the corks considered by Akbulut and Yasui in [AY08] generate the $\mathbb{Z}^\infty$-subgroup of Theorem 3.2. Previously it was shown by Lin, Ruberman, and Saveliev that the original Akbulut-Mazur cork [Akb91] is strong.

Question 3.3. In [DHM20], we showed that many known examples of corks were strong, and in fact conjectured that *all* existing examples of corks in the literature are strong. In [HP20], Hayden and Piccirillo gave an example of a (new) cork which was not strong, answering [DHM20, Question 1.14]. Are there any deeper topological consequences or characterizations of a cork being strong? For example, in the cork twist theorem of Matveyev [Mat96] and Curtis, Freedman, Hsiang, and Stong [CFHS96], can the cork be taken to be strong?

[2] That is, if Y is a 3-manifold with diffeomorphism f, then there exists some W over which f extends.

Question 3.4. Can [DHM20] or [DMS21] be extended to study higher-order diffeomorphisms? What about orientation-reversing involutions on 3-manifolds? Are there interesting interactions between our Floer-theoretic methods and infinite-order corks?

Question 3.5. In many cases, the nontriviality of the action of τ on the Floer homology of Y is related to the relevant cork twist changing the Seiberg-Witten invariant. Find effective embeddings of different families of strong corks. (That is, embeddings in which the cork twist changes the smooth structure.) Can one find a general construction in certain cases (for example, surgeries on equivariant slice knots)?

4. Equivariant Knots

Broad Goals. Use knot Floer homology to study equivariant sliceness and equivariant genus. What can this formalism say about exotic pairs of surfaces in the context of symmetric knots?

In work with Mallick and Stoffregen, I extend the approach in [DHM20] to the setting of equivariant knots. In [DMS21], we define the following:

Theorem 4.1. [DMS21, Theorem 1.1, Theorem 1.2] *Let (K, τ) be a strongly invertible or 2-periodic knot in S^3. Associated to (K, τ) we have four integer-valued concordance invariants*

$$\overline{V}_0^{\tau}(K) \leq \underline{V}_0^{\tau}(K) \quad \text{and} \quad \overline{V}_0^{\iota\tau}(K) \leq \underline{V}_0^{\iota\tau}(K)$$

which vanish if K is equivariantly slice. Moreover, appropriate functions of these give a lower bound for the equivariant slice genus $\widetilde{g}_4(K)$.

The invariants of Theorem 4.1 are constructed by considering the action of τ on the knot Floer complex of K.

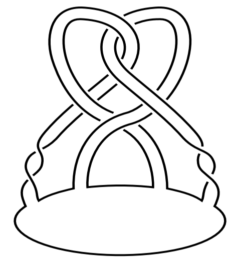
Equivariant sliceness has been studied by several authors, but bounding the equivariant slice genus has only been investigated more recently. In [BI21], Boyle and Issa exhibited families of 2-periodic knots for which $\widetilde{g}_4(K) - g_4(K)$ grows arbitrarily large, and conjectured the same should hold for strongly invertible knots. We use our invariants to provide an affirmative answer to this:

Theorem 4.2. [DMS21, Theorem 1.4] *There exist families of strongly invertible slice knots whose equivariant slice genus grows arbitrarily large.*

In fact, Theorem 4.1 bounds a rather stronger quantity that the equivariant genus. We say that a slice surface Σ for K is *isotopy equivariant* if its image $\tau(\Sigma)$ (under some smooth extension of τ over B^4) is isotopic to Σ rel boundary. In [DMS21], we define the *isotopy equivariant slice genus* $\widetilde{ig}_4(K)$ of K to be the minimal genus over all such Σ. Although the notion of isotopy equivariance may initially seem rather contrived, a slight shift in perspective demonstrates its usefulness. Indeed, if Σ is any slice surface for K with $g(\Sigma) < \widetilde{ig}_4(K)$, then we may immediately conclude that the two surfaces Σ and $\tau(\Sigma)$ are not isotopic rel K. The calculation of $\widetilde{ig}_4(K)$ thus provides an easy method for generating non-isotopic slice surfaces in the presence of a symmetry on K.

For example, if K is an equivariant slice knot with $\widetilde{ig}_4(K) > 0$, then we may take *any* slice disk Σ for K and form its image under any extension of τ; the resulting pair of slice disks are automatically non-isotopic rel K. This is in marked contrast to the usual approach taken in the literature, where in order to deploy various invariants, one has in mind a specific family of slice disks or surfaces that are conjectured to be non-isotopic. The situation here is analogous to that of a strong cork from [LRS18]. In fact, one can further upgrade the notion of isotopy equivariant genus by allowing any homology ball in place of B^4; the bounds of Theorem 4.1 still hold. Using Theorem 4.1, we were able to study (among other examples) the following knot, first due to Hayden [Hay21]:

Theorem 4.3. [DMS21, Theorem 1.5. Theorem 1.6] *The knot K displayed below has $\widetilde{ig}_4(K) > 0$. Thus, if Σ is any slice disk for K, then Σ and $\tau(\Sigma)$ are not isotopic rel boundary. Moreover, this fact is detected by knot Floer homology, in the sense that the two induced maps F_Σ and $F_{\tau(\Sigma)}$ on knot Floer homology differ.*



In [Hay21], Hayden showed that K admits a particular pair of slice disks which are topologically but not smoothly isotopic rel boundary; this pair is in fact related by a symmetry τ. Theorem 4.3 thus provides a Floer-theoretic re-proof of Hayden's result. (The fact that the disks are topologically isotopic is immediate from the work of Conway and Powell [CP19], using the fact that the disks have fundamental group $\mathbb{Z}$.) Knot Floer homology has previously been used to detect exotic higher-genus surfaces (see the work of Miller, Juhász, and Zemke [JMZ20]), but Theorem 4.3 gives the first example of the use of knot Floer homology in detecting an exotic pair of disks.

Question 4.4. In [HS21], Hayden and Sundberg used Khovanov homology to obtain an alternative proof of the fact that K admits a pair of exotic slice disks. This suggests that one should be able to construct an analogous formalism to the one in [DMS21] on the Khovanov side. Together with Borodzik, Mallick, and Stoffregen, I am currently pursuing this line of research.

5. Lattice Homology and Combinatorial Aspects

Broad Goals. Develop effective ways of computing Floer homology. Understand the relationship between different Floer theories and combinatorial models.

I have also written several papers on the connection between *lattice homology* and various Floer theories. Lattice homology is a combinatorial invariant for plumbed manifolds due to Ozsváth and Szabó [OS03], which is conjecturally isomorphic to Heegaard Floer homology. This has been verified for a large class plumbed manifolds (for example, Seifert fibered spaces), and in general it is known that there is a spectral sequence relating the two [OSS14]. In joint work with Alfieri, Baldwin, and Sivek, I showed that (in certain cases) lattice homology could also be used to compute the framed instanton Floer homology of Kronheimer and Mrowka [KM11]. As a result, we obtained:

Theorem 5.1. [ABDS20, Corollary 1.3] *Let Y be a Seifert fibered rational homology sphere (or, more generally, an almost-rational plumbed manifold). Then*

$$I^{\#}(Y) \cong \widehat{HF}(Y;\mathbb{C})$$

as $\mathbb{Z}/2\mathbb{Z}$-graded vector spaces.

Theorem 5.1 is important in light of the conjecture that Heegaard Floer and instanton Floer homology are isomorphic in all cases; see [KM10, Conjecture 7.24].

More generally, lattice homology provides a useful tool for computing various Floer homologies. In joint work with Manolescu [DM19], I showed that lattice homology could be used to compute the involutive Heegaard Floer homology of Seifert fibered spaces, following an earlier work in which I showed that the Pin(2)-equivariant monopole Floer homology could similarly be computed [Dai18]. (See also the work of Stoffregen [Sto20].) Techniques from lattice homology can also be used to efficiently extract information about connected sums of Seifert fibered spaces. In joint work with Stofferegen, I used this to show:

Theorem 5.2. [DS19, Corollary 1.3, Corollary 1.4] *The Neumann-Siebenmann invariant $\bar{\mu}$ descends to a well-defined homomorphism*

$$\bar{\mu}\colon \Theta_{SF} \to \mathbb{Z},$$

where Θ_{SF} is the subgroup of $\Theta^3_{\mathbb{Z}}$ generated by Seifert fibered spaces. In particular, if Y is a linear combination of Seifert fibered spaces with Rokhlin invariant one, then $[Y]$ is not torsion in $\Theta^3_{\mathbb{Z}}$.

Theorem 5.2 is also important in light of the conjecture that $\bar{\mu}$ is a homology cobordism invariant for all plumbed 3-manifolds. I used similar techniques in [Dai19] to compute the *connected Floer* *homology* of linear combinations of Seifert fibered spaces (see [HHL21]). The computations of [DM19], [DS19], and [Dai19] were incorporated into [DHST18]; this was later used by Hendricks, Hom, Stoffregen, and Zemke to show that $\Theta^3_{\mathbb{Z}}$ is not generated by Seifert fibered spaces [HHSZ20].

Lattice homology also constitutes a bridge between Floer theory and algebraic geometry. In [Ném08], Némethi used lattice homology to study the algebraic geometry of certain surface singularities. This is especially interesting in light of the work of Mrowka, Oszváth, and Yu [MOY96], who showed that for Seifert fibered spaces, the different moduli spaces of flow lines in monopole Floer homology similarly have an interpretation in terms of the algebraic geometry of an associated ruled surface. Némethi also showed that lattice homology can be expressed in terms of the homology of a particular cubical complex, in a setup analogous to that of persistent homology or discrete Morse theory. This connection naturally leads to the following long-term question:

Question 5.3. Can lattice homology be used to compute the Seiberg-Witten-Floer spectrum of Seifert fibered spaces? Can we use lattice homology to define a Heegaard Floer homotopy type (at least for plumbed 3-manifolds)?

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