1. (6 points) Solve §1.3 (page 79), problem 9.

2. (7 points) Finding a covering space corresponding to a particular subgroup. Hatcher motivates §1.3 by explicitly discussing a series of examples of covering spaces of $S^1 \lor S^1$. Read this motivation, and then solve §1.3 (page 80), problem 12.

3. (10 points) Explicitly classifying covering spaces. Solve §1.3 (page 80), problem 14. It may be helpful to read about Cayley graphs and complexes on pages 77-78, as a method of explicitly constructing the universal covers of some spaces—in particular, Example 1.48 gives a construction of the universal cover of $\mathbb{R}P^n \lor \mathbb{R}P^n$.

4. (8 points) Covering spaces from group actions. Given a group $G$ and a space $Y$, an action of $G$ on $Y$ is a homomorphism $\rho$ from $G$ to $\text{Homeo}(Y)$ the group of homeomorphisms from $Y$ to $Y$ (multiplication is composition). Thus, for $g \in G$, $\rho(g) : Y \to Y$ is a homeomorphism and $\rho(g \cdot g') = \rho(g) \circ \rho(g')$. For a fixed group action, $\rho$ is often implicit, and instead we write $g : Y \to Y$ for $g \in G$. Such an action $\rho$ is called a covering space action if for each $y \in Y$, there exists a neighborhood $U \subset Y$ containing $y$ such that the collection $\{g(U) | g \in G\}$ is a disjoint collection (i.e. $g_1(U) \subset g_2(U) \neq \emptyset \Rightarrow g_1 = g_2$).

Given any action of $G$ on $Y$, one can form the orbit space (or quotient)
\[
Y/G := Y/\sim \quad \text{where } y \sim g(y) \text{ for all } g \in G.
\]

If $G$ is a covering space action, then by Proposition 1.40 in Hatcher, then the projection $Y \to Y/G$ is a normal covering space, and moreover $G$ is the group of deck transformations of the covering space (if $Y$ is path-connected).

A first example is given by the $\mathbb{Z}/2\mathbb{Z}$ action on $S^n$ generated by the antipodal map. The quotient space is $\mathbb{R}P^n$, which we defined in class. More generally, nice group actions give us many more examples of covering spaces. (For example, take $Y = S^{2n-1}$, thought of as a subspace of $\mathbb{C}^n$, and the $\mathbb{Z}/k\mathbb{Z}$ action generated by $z \mapsto ze^{2\pi i/k}$. The orbit space is an example of a lens space, and has fundamental group $\mathbb{Z}/k\mathbb{Z}$ (when $n > 1$).

Read the section in Hatcher on covering space actions (page 71-74). Then solve §1.3 (page 81), problem 23.

5. (8 points) An example of a covering space action. Solve §1.3 (page 81), problem 25.