A quantitative theory of stochastic homogenization

Felix Otto, Max Planck Institute, Leipzig, Germany

In many applications, one has to solve an elliptic equation with coefficients that vary on a length scale much smaller than the domain size. We are interested in a situation where the coefficients are characterized in statistical terms: Their statistics are assumed to be translation invariant (i. e. stationary) and to decorrelate over large distances. In this situation, the solution operator behaves – on large scales – like the solution operator of an elliptic problem with *homogeneous, deterministic* coefficients! Qualitatively, this is by now well-understood under mild conditions (stationarity and ergodicity) and in very general situations (e. g. in case of percolation).

This course about a *quantitative* theory: We present elements of a calculus that allow to infer more quantitative statements (i. e. the size of various homogenization errors) under more quantitative assumptions on the decorrelation. It turns out that notions from statistical mechanics, like the Spectral Gap Inequality and the Logarithmic Sobolev Inequality, are ideally suited to quantify decorrelation. The calculus combines this with classical estimates from regularity theory: both parabolic regularity theory (Nash) and elliptic regularity theory (de Giorgi).

The course relies on joint work with A. Gloria, S. Neukamm, and D. Marahrens. In particular, we refer to the three preprints, which are available on my web page: [3] requires the least machinery and will be closest to the course, [1] gives an extensive introduction next a couple of quantitative results, and [2] uses both to give a full error estimate.

References

[1] Antoine Gloria, Stefan Neukamm, and Felix Otto. Quantification of ergodicity in stochastic homogenization: optimal bounds via spectral gap on Glauber dynamics - long version.

[2] Antoine Gloria, Stefan Neukamm, and Felix Otto. An optimal quantitative two-scale expansion in stochastic homogenization of discrete elliptic equations.

[3] Daniel Marahrens and Felix Otto, Annealed estimates on the Greens function.