18.014 PRACTICE QUIZ III

You may use a crib sheet (one side of an 8 1/2 by 11 sheet of papers, but no calculator. Justify all steps — if in doubt, please ask. Time: 50 minutes.

1. (28 points) Evaluate:

(a) $\int \frac{x\,dx}{\sqrt{2x+1}}$.

(b) $\int_e^{e^2} \frac{dx}{x \ln x}$.

(c) $\int \frac{x\,dx}{x^2+4x+6}$.

(d) $\int \frac{x+1}{(x^2+1)^2}\,dx$.

2. (10 points) Derive a recursion formula for $\int x^n e^{ax}\,dx$ in terms of $\int x^{n-1} e^{ax}\,dx$.

3. (14 points) For what value of n is the following limit finite and nonzero?

$$\lim_{x \to 0} \frac{(\sin x)(x - \sin x)(e^{5x} - 1)}{x^n}$$

What is this limit?

4. (20 points) (a) Use the second order Taylor approximation to e^x near $x = 0$ to compute (approximately) the integral

$$\int_0^{1/2} e^{(x^2)}\,dx.$$

(Leave your answer as a sum of fractions.)

(b) Obtain an upper bound on the error, given that $e^{1/4} < 4/3$. (Leave your answer as a fraction.)

5. (28 points) Evaluate

(a) $\lim_{x \to 0^+} (\ln(3x + e))^{1/x}$.

(b) $\lim_{x \to +\infty} (\ln(3x + e))^{1/x}$.

(c) $\lim_{x \to +\infty} \frac{\cos(1/x)}{\arctan x}$.

(c) $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{e^x - 1}\right)$.

Challenge problem. Let N be the positive integer with 1998 decimal digits, all of them 1; that is,

$$N = 111 \cdots 11.$$

Find the thousandth digit after the decimal point of $\sqrt{N}$.

(There won't be a challenge problem on the quiz itself. If you are timing yourself with this practice quiz, don't try to do the challenge problem in the allotted time.)