18.014 PRACTICE QUIZ IV

Time: 60 minutes (no crib sheet). There are questions on both sides of the sheet. The actual quiz will take place on Tuesday, Dec. 19 from 10 to 11 in 4–159. The best preparation for it will include knowing the answers to these questions and to the problem set problems very well.

1. (20 points) Find the intervals of convergence of the following power series. (Don't bother with the endpoints.)

(a) $\sum_{n=1}^{\infty} \frac{(x+1)^n}{n2^n}$. (b) $\sum_{n=1}^{\infty} \frac{n^n}{n!} x^n$.

2. (25 points) Test for convergence; given reasons for your answer.

(a) $\sum_{n=1}^{\infty} \sqrt{n} \sin^2(1/n)$. (b) $\sum_{n=1}^{\infty} (1 - \sin \sqrt[n]{n})^n$. (c) $\sum_{n=2}^{\infty} \frac{1}{(\ln n)^{100}}$.

3. (10 points) Determine the radius of convergence of the following power series:

$$\sum_{n=1}^{\infty} \frac{n^{\sqrt{n}}}{2^n} x^n.$$

(Show your work.)

4. (15 points) Use the integral test to obtain upper and lower bounds for the number

$$E = \left(\sum_{n=1}^{\infty} \frac{1}{n^3} \right) - \left(1 + \frac{1}{8} + \frac{1}{27} \right).$$

Leave answers as fractions.

5. (15 points)

(a) Find the radius of convergence of the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{x^{3n}}{(3n)!}.$$

Prove that $f'''(x) = f(x)$ on the interval of convergence.

(b) Show that there is a solution $g(x)$ (defined for all real numbers x) to the differential equation $g'''(x) = g(x)$ satisfying $g(0) = g'(0) = 0$, $g''(0) = 1$. (Hint: look at (a).)

Date: Fall 2000.

Comment for the experts: the function in (a) is actually

$$\frac{1}{3}\left(e^x + 2e^{-x/2}\cos\left(\sqrt{3}x/2\right)\right).$$

6. (15 points) Suppose the series $\sum_{n=0}^{\infty} a_n x^n$ converges for $x = -3$.

(a) What can you say about the radius of convergence of the series?

(b) Does it follow that the series $\sum_{n=1}^{\infty} |a_n| n 2^n$ converges? Explain your answer.

The following question is also good practice; it could be substituted for 4 or 5 or 6 above.

***.** (15 points) Suppose $\sum |a_n|$ converges.

(a) Does it follow that $\sum (a_n)^2$ converges? Diverges?

(b) Does it follow that $\sum \left(\frac{a_n - 1}{n}\right)$ converges? Diverges?

Give a reason for each answer.