18.014 QUIZ I

If you have any questions, please ask. Hint: don't feel compelled to work through the problems in order!

1. (16 points)

(a) State the triangle inequality for $|a + b|$.
(b) Show that $|x| - |y| \le |x - y|$ for all x, y.

2. (16 points) State the Riemann condition for the existence of the integral $\int_a^b f$, where f is a function on $[a, b]$.

3. (16 points) Evaluate $\int_{-1}^{2} x^2[x]dx$, where $[\cdot]$ denotes the "greatest integer" function.

4. (16 points) Suppose that $\int_0^1 \frac{x}{x^6+1}dx = a$ and $\int_0^2 \frac{x}{x^6+1}dx = b$. Express $\int_{-2}^{-1} \frac{3x}{x^6+1}$ in terms of a and b.

5. (16 points) Consider the solid in three-space that lies above $z = 0$, such that the cross-section for given z is a square with sides parallel to the x and y axes having as left edge the line segment connecting the point $(z, 0)$ on the x-axis to the point (z, z^3) on the curve $y = x^3$. Find the volume of the portion of the solid between $z = 0$ and $z = a$, where $a > 0$.

6. (20 points) Suppose x and y are real numbers with $x < y$.

(a) If $y - x > 1$, show that there is an integer z such that $x < z < y$. (You may use standard properties of the integers. If you use the well-ordering principle, the Archimedean property, or the principle of induction, mention the fact that you are using it.)
(b) Even if $y - x$ is not greater than 1, show that there is a rational number r such that $x < r < y$. (Hint: Why is there a positive integer n such that $y - x > 1/n$? Then consider $nx < ny$ instead of $x < y$.)

GOOD LUCK!

Date: Fall 2000.