18.014 SAMPLE QUIZ I

1. (16 points) Let a be a positive real number that is *not* an integer. Show there exists an integer n such that $a < n < a+1$. (You may use standard properties of the integers. If you use the well-ordering principle, the Archimedean property, or the principle of induction, mention the fact that you are using it.)

2. (16 points) Let f be a bounded function on the interval $[a, b]$. State the Riemann condition for the existence of the integral $\int_a^b f$.

3. (16 points) Let s be a step function on $[a, b]$. Suppose $x_0 < x_1 < \cdots < x_n$ is a partition of $[a, b]$ such that $s(x) = s_k$ for $x_{k-1} < x < x_k$. Suppose we define $\int_a^b s = \sum_{k=1}^{n} (s_k)^2 (x_k - x_{k-1})$. This integral is well-defined. Which of the following properties hold, and which fail? (Just give answers, not reasons; 4 points for each correct answer, and -2 for each wrong answer.)

(a) $\int_a^b cs = c \int_a^b s$, c constant.
(b) $\int_a^b (s+t) = \int_a^b s + \int_a^b t$.
(c) $\int_a^b s + \int_b^c s = \int_a^c s$ if $a < b < c$.
(d) $\int_a^b s \leq \int_a^b t$ if $s \leq t$ on $[a, b]$.

4. (21 points) Evaluate:

(a) $\int_0^2 [2x] dx$.
(b) $\int_0^2 2[x] dx$.
(c) $\int_0^2 |x^2 - 1| dx$.

5. (21 points) Consider the region bounded above by $y = 1 - x^2$ and below by the x-axis. Revolve it about the horizontal line $y = 1$. Find the volume thus generated.

6. (10 points) Consider the following cases: *(i)* x, y both rational; *(ii)* x, y both irrational; *(iii)* x rational and y irrational.

(a) In which of these cases does it follow that $x + y$ is rational?
(b) In which of these cases does it follow that $x + y$ is irrational?

(Give answers only.)

Date: Fall 2000.