18.024 QUIZ I

Write your name on the first page of your solutions. Time: 50 minutes. Justify all steps. If you have any questions, please ask. **GOOD LUCK!**

1. (18 points) Let W be a subspace of V_n.

(a) Define what it means for the vectors $A_1, \ldots, A_k$ of W to *span* W.
(b) Define what it means for the vectors $B_1, \ldots, B_m$ of W to be *linear independent*.
(c) If the vectors $A_1, \ldots, A_k$ of W span W, and the vectors $B_1, \ldots, B_m$ of W are linearly independent, what is the relation between k and m and $\dim W$? (Answer only; no explanation required.)

2. (18 points) Let W be the subspace of V_4 spanned by $(1, -1, 1, 1)$, $(2, 0, -1, 1)$, and $(1, 1, -2, 0)$.

(a) What is the dimension of W?
(b) Find a basis for the orthogonal complement of W.

3. (16 points) Find parametric equations for the line in V_3 through the point $P = (-1, 2, 1)$ that is parallel to the line of intersection of the plane $3x - y + z = 1$ and $x + 2z = 4$.

4. (16 points) Consider the vectors $A = (1, -1, 2, 3)$ and $B = (0, 1, 1, 1)$ of V. One can write $A = tB + C$, where tB is parallel to B, and C is perpendicular to B. Find t and C.

5. (16 points) Find the inverse of the matrix

$$\begin{pmatrix} 1 & -1 & -1 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

6. (16 points) If

$$\begin{pmatrix} 1 & 0 & 0 & 7 \\ -1 & 2 & 2 & 3 \\ 1 & 0 & 1 & 2 \\ -1 & 0 & 0 & 1 \end{pmatrix} B = \begin{pmatrix} 0 & 0 & 0 & 5 \\ 1 & 2 & -1 & 1 \\ 0 & 2 & 1 & 2 \\ 0 & 0 & 3 & 6 \end{pmatrix},$$

find $\det B$.

Date: Spring 2001.