18.034 PROBLEM SET 3

Solve eight of the following problems. Due February 25 in class. No lates will be accepted. Discussion is encouraged, with two caveats: (a) write up your solutions by yourself, and (b) give credit when others came up with ideas (you won't be penalized for this). Give explanations, not just answers. References are to Boyce and DiPrima; the answers to problems from the book are in the back of the book.

1. Find a formula for the nth Fibonacci number F_n. (The Fibonacci numbers are defined by $F_0 = 0$, $F_1 = 1$, $F_{n+2} = F_{n+1} + F_n$.)
2. (a) Problem 3.6.1. (b) Problem 3.6.3. (c) Problem 3.6.6.
3. (a) Problem 3.6.8. (b) Problem 3.6.9. (c) Problem 3.6.27.
4. (a) Check that $\int (x^2+1)e^x dx$ is of the form $g(x)e^x + C$ where $\deg g = 2$.
   (b) Prove that $[D, x^n] = nx^{n-1}$ and $[D^m, x] = mD^{m-1}$ for positive integers m and n.
5. (a) Problem 3.1.28. (b) Problem 3.1.30.
6. (a) Problem 3.1.34. (b) Problem 3.1.36.
7. *Orthogonal trajectories.* Problem 2.10.37 (a)-(c).
8. *Application: Mechanical and electrical vibrations.* Skim Section 3.8 (and note the definitions). Solve problems 3.8.1, 3.8.5, and 3.8.19.
9. *Complex numbers and geometry.* Prove the following theorem in plane geometry using complex numbers: *Suppose $ABCD$ is a convex quadrilateral. Construct squares outwards on the sides of $ABCD$, and let N, O, P, Q be their centers (as in the figure). Prove that $NP = OQ$ and that NP and OQ are perpendicular.*

   Here's how. Place the figure in the complex plane. Let A be the complex number corresponding to the point A (and similarly for points b, ..., p). Then the vector $\vec{AB}$ is $b - a$.
   (a) Explain why $(f - b) = i(a - b)$. Hence solve for f in terms of a and b.
   (b) N is the midpoint of AF. Hence solve for n in terms of a and b using $n = (a + f)/2$.
   (c) Similarly, solve for o in terms of b and c, p in terms of c and d, and q in terms of d and a.
   (d) Show that $(p - n) = i(o - q)$. Why does this prove the theorem?

Date: February 18, 2000.