18.034 PROBLEM SET 7

Due April 7 in class. No lates will be accepted. Discussion is encouraged, with two caveats: (a) write up your solutions by yourself, and (b) give credit when others came up with ideas (you won't be penalized for this). Give explanations, not just answers. References are to Boyce and DiPrima; the answers to problems from the book are in the back of the book.

1. *Transforming an initial value problem into an equivalent problem with initial point at the origin.* Problem 2.11.1, p. 106.
2. *An example of how limits of integrals isn't necessarily the integral of limits.* Problem 2.11.14, p. 106–107.
3. Suppose your calculator has a button which sends x to $1 + 1/x$. If you input the number 1, and then hit the button many times, what number would the iterations converge to? (This is the *golden mean*, which comes up in Fibonacci numbers, and in nature in the geometry of sunflowers and pineapples.) Prove this! Hint: use the contraction mapping theorem, and the interval $[3/2, 2]$. Warning: 1 isn't in this interval!
4. Consider the differential equation $y' = -y + 1$, $y(0) = 0$. Write down the corresponding integral equation. Write a formula for the nth Picard iterate. (Hint: calculate the first four Picard iterates to look for the pattern. Solving the differential equation may also give you an idea.)
5. (a) By computing the appropriate Lipschitz constant, show that the function $f(x, y) = 4x^2 + y^2$ satisfies a Lipschitz condition on the rectangle given by $|x| \leq 1$, $|y| \leq 1$. (There is a little bit on Lipschitz conditions in the text, see Problem 2.11.13.)
   (b) Show that the function $f(x, y) = \sqrt{y}$ does not satisfy a Lipschitz condition on $|x| \leq 1$, $0 \leq y \leq 1$.

Date: March 31, 2000.