MATH 121 PROBLEM SET 1

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This set is due at 6 p.m. on Monday January 29 in Jason Lo's mailbox.

1. Show that $\mathbb{Z}[i]/(7)$ is a field. Show that $\mathbb{Z}[i]/(5)$ is not a field. Show that $\mathbb{Z}[i]/(2)$ is not a field.

2. Show that $p(x) = x^3 + 9x + 6$ is irreducible in $\mathbb{Q}[x]$. Let α be a root of $p(x)$. Find the inverse of $1 + \alpha$ in $\mathbb{Q}(\alpha)$ (in the form $?+?\alpha+?\alpha^2$).

3. Prove that if $[F(\alpha) : F]$ is odd then $F(\alpha) = F(\alpha^2)$.

4. Suppose $F = \mathbb{Q}(\alpha_1, \alpha_2, \dots, \alpha_n)$ where $\alpha_i^3 \in \mathbb{Q}$ for $i = 1, 2, \dots, n$. Prove that $\sqrt[4]{2} \notin F$.

5. Suppose E/F is a field extension, and E_1 and E_2 are two subextensions. Show that if E_1 and E_2 are finite (over F) then their compositum is finite. Show that if E_1 and E_2 are algebraic then their compositum is algebraic.

6. Find all intermediate fields in $\mathbb{Q}(\sqrt{2}, \sqrt{3})/\mathbb{Q}$. Find the automorphism group of this field extension. (An automorphism of a field extension E/F is a field automorphism of E that preserves all the elements of the subfield F.) Find all α such that $\mathbb{Q}(\alpha) = \mathbb{Q}(\sqrt{2}, \sqrt{3})$.

Date: Monday, January 22, 2007. *Last updated Jan. 25.*