MATH 210 PRACTICE MIDTERM

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Justify all answers!

1. (a) Show that $\mathbb{Q}(\sqrt{2})$ and $\mathbb{Q}(\sqrt{3})$ are not isomorphic fields.

(b) Show that $\mathbb{Q}(\sqrt{\pi})$ and $\mathbb{Q}(\pi)$ are isomorphic fields. (You may use the fact that π is transcendental.)

2. Show that $\mathbb{Q}(\sqrt{2+\sqrt{2}})$ is a cyclic quartic field, i.e. is a Galois extension of $\mathbb{Q}$ of degree 4 with cyclic Galois group.

3. Show that $\mathbb{Q}(x_1, x_2, x_3)/\mathbb{Q}(e_1, e_2, e_3, (x_1 - x_2)(x_1 - x_3)(x_2 - x_3))$ is a Galois extension, with Galois group of order 3.

4. Suppose $p(x)$ be an irreducible polynomial in $\mathbb{Q}[x]$ whose splitting field has cyclic Galois group. Show that $p(x)$ must have all real roots or all non-real roots.

5. Suppose E/F is an algebraic field extension. Show that E/F is the splitting field of a family of polynomials if and only if every irreducible polynomial in $F[x]$ with a root in E splits completely. If E/F is finite, show that these two are equivalent to the statement that any map from E to $\overline{F}$ fixing F must send E to itself.

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