MATH 210 PROBLEM SET 1

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This problem set is due on Monday, January 29 at Jarod Alper's office door.

1. Suppose ω is a nontrivial cube root of 1, and $\overline{\omega}$ is its conjugate (another cube root of 1). Show that $\overline{\omega}\sqrt[3]{2} \notin \mathbb{Q}(\omega\sqrt[3]{2})$.

2. Give an example (with proof!) of a quadratic field extension E/F that is not obtained by adjoining a square root of F.

3. Suppose E/F is a field extension, and E_1 and E_2 are two subextensions. Show that if E_1 and E_2 are finite (over F) then their compositum is finite. Show that if E_1 and E_2 are algebraic then their compositum is algebraic.

4. Find all intermediate fields in $\mathbb{Q}(\sqrt{2}, \sqrt{3})/\mathbb{Q}$. Find the automorphism group of this field extension. Find all α such that $\mathbb{Q}(\alpha) = \mathbb{Q}(\sqrt{2}, \sqrt{3})$.

5. How many irreducible monic degree 10 polynomials are there over $\mathbb{F}_p$?

6. Show that the degree of the splitting field of $x^3 - 3x + 1$ is 3. (Please explain where your ideas came from — don't just pull a random expression out of your hat!)

Date: Monday, January 22, 2007.