MATH 210 PROBLEM SET 7

RAVI VAKIL

This problem set is due on Friday, March 16 at Jarod Alper's office door.

1. Suppose that A and B are ideals with $AB \subset Q$ for a primary ideal Q. Prove that if A is not contained in Q, then $B \subset \sqrt{Q}$. (Dummit and Foote 15.2 problem 29)

2. Show that the intersection of two P-primary ideals of a ring R is also P-primary. (Dummit and Foote 15.2 problem 31)

3. Prove that a prime ideal P contains the ideal I if and only if P contains one of the associated primes of a minmial primary decomposition of I. (Dummit and Foote 15.2 problem 37)

4. Let $P_1, \ldots, P_m$ be the associated prime ideals of the ideal (0) in the Noetherian ring R.

(a) Show that $P_1 \cap \cdots \cap P_m$ is the collection of nilpotent elements in R.

(b) Show that $P_1 \cup \cdots \cup P_m$ is the collection of zero divisors in R.

(Dummit and Foote 15.2 problem 41; some hints are given there. Caution: if you use Corollary 22 from the book, you'll have to prove it, as we haven't done it in class.)

5. Prove that the ideal I in the Noetherian ring R is radical if and only if the primary components of a minimal primary decomposition are all prime ideals, and conclude that in this case the minimal primary decomposition is unique. (Dummit and Foote 15.2 problem 43; some hints are given there.)

6. Describe the Zariski topology on $\operatorname{Spec} \mathbb{C}[t]$.

Date: Friday, March 9, 2007.