MATH 120 PRACTICE MIDTERM

Give complete proofs unless otherwise indicated.

1. *(6 points)* For this question, give answers only.

(a) Give a Jordan-Holder decomposition of S_3. In other words, give a nested sequence of normal subgroups, where the quotient of each by the next smaller one is simple.

(b) What is the center of S_3?

2. *(6 points)* The group $SL(2, \mathbb{Z})$ consists of those 2×2 matrices with integer entries and determinant 1. Let N be the subset consisting of matrices congruent to the identity matrices modulo 2, i.e. with odd diagonal entries and even off-diagonal entries. Show that N is a normal subgroup of $SL(2, \mathbb{Z})$. (Hint: describe N as the kernel of the map from $SL(2, \mathbb{Z})$ to another group.)

3. *(6 points)* Suppose G acts on a set A, and a and $b \in A$ are in the same orbit of G. Show that G_a, the stabilizer of a, is conjugate to G_b, the stabilizer of b.

4. *(6 points)* Describe a bijection between the conjugates of an element g of a group G and the cosets of the centralizer of g (consisting of those elements s of G such that $gs = sg$).

5. *(6 points)* Suppose both H and K are normal subgroups of G with $H \cap K = \{e\}$. Show that $xy = yx$ for all $x \in H$ and $y \in K$. (Hint: move everything to one side of the equation, and show that the result is e.)

Date: October 21, 2009.