Propagation of singularities in three-body scattering

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Abstract

In this thesis we consider a compact manifold with boundary X equipped with a scattering metric g and with a collection C_i of disjoint closed embedded submanifolds of ∂X. Thus, g is a Riemannian metric in $\mathrm{int}(X)$ of the form $g = x^{-4}\,dx^2 + x^{-2}h$ near ∂X for some choice of a boundary defining function x, h being a smooth symmetric 2-cotensor on X which is non-degenerate when restricted to ∂X. We also let Δ be the (positive) Laplacian of g, suppose that $V \in \mathcal{C}^\infty([X;\cup_i C_i])$ where $[X;\cup_i C_i]$ is X blown up along the C_i, assume that V vanishes at the lift of ∂X, and consider the operator $H = \Delta + V$. Three-body scattering with smooth potentials which have an asymptotic expansion at infinity (possibly Coulomb-type) provide the standard example of this setup. We analyze the propagation of singularities of generalized eigenfunctions of H, showing that this is essentially a hyperbolic problem which has much in common with the Dirichlet and transmission problems for the wave operator, though additional features arise due to the presence of bound states of the 'two-body operators'. We also show that the wave front relation of the free-to-free part of the scattering matrix is given by the broken geodesic flow at distance π.

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