ERRATUM TO 'PROPAGATION OF SINGULARITIES IN MANY-BODY SCATTERING IN THE PRESENCE OF BOUND STATES'

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The proof of Theorem 2.2 in [2, pp. 241-243] is incomplete, since Proposition 7.1[1] is not stated (and proved) in a sufficiently strong form. Using the notation of [2, Section 7], the reason is that in (i) of Proposition 7.1, $\gamma : s \mapsto \exp(s\,{}^{\mathrm{sc}}H_g^a)(\bar{\xi})$ is not necessarily a generalized broken bicharacteristic. In fact, writing $\bar{\xi} = (\bar{y}_a, \bar{\tau}_a, \bar{\mu}_a) \in {}^{\mathrm{sc}}T^*_{C'_a}\bar{X}_a$, γ is a generalized broken bicharacteristic if and only if for some subsystem b, $(\bar{y}_a, 0, \bar{\tau}_a, \bar{\mu}_a, 0) \in \Sigma_b(\lambda)$, where the two zeros refer to the coordinates z_a and ν_{ab}, respectively. (The 'if' part is clear, in fact γ is then an integral curve of ${}^{\mathrm{sc}}H_g^b$ in the b-characteristic set $\Sigma_b(\lambda)$ which happens to be tangential to, hence lie over, the cluster C_a, and for the 'only if' it suffices to consider the π-invariant function $\eta_a = z_a \cdot \nu_a$, as this would be strictly increasing along γ otherwise, due to Definition 2.1; see (3) below.)

Thus, in part (i) of Proposition 7.1, there should be two cases, depending on whether $(\bar{y}_a, 0, \bar{\tau}_a, \bar{\mu}_a, 0) \in \Sigma_b(\lambda)$ for some b, or not. If $\bar{\mu}_a = 0$, automatically the latter is the case (since $\bar{\xi}$ is not radial, i.e. $\bar{\xi} \in S$, by assumption), so in fact (ii) of Proposition 7.1 can be taken to be a part of the latter case. Now, the statement of the proposition is optimal in the former case, but not in the latter. However, in the latter case the argument provided for $\bar{\mu}_a = 0$ can be easily modified to complete the proof, roughly along the lines of the proof of the normal propagation result in [1, Proposition 10.1] (where the absence of bound states in proper subsystems was assumed).

We now state this corrected version, which completes the proof of Theorem 2.2 by the argument on pp. 241-243.

Proposition 1. *[Improving Proposition 7.1 of [2]] Let H be a many-body Hamiltonian on $(\bar{X}, \mathcal{C})$, and let $u \in \mathcal{C}^{-\infty}(\bar{X})$ be a microlocal eigenfunction of H at a non-radial point $\bar{\xi}$ over the regular part of the cluster C_a, that is $\bar{\xi} = (\bar{y}_a, \bar{\tau}_a, \bar{\mu}_a) \in S \cap {}^{\mathrm{sc}}T^*_{C'_a}\bar{X}_a$, and $\bar{\xi} \notin \mathrm{WF}_{Sc}((H-\lambda)u)$. Let $\eta_a = z_a \cdot \nu_a$. Then there exists a neighborhood U_0 of $\bar{\xi}$ with the following properties.*

(i) *(Tangential propagation is possible.) Suppose $(\bar{y}_a, 0, \bar{\tau}_a, \bar{\mu}_a, 0) \in \Sigma_b(\lambda)$ for some b. If there exists a neighborhood $U \subset U_0$ of $\bar{\xi}$ in $\dot{\Sigma}(\lambda)$, and there exists $s_0 > 0$ such that $\exp(-s_0\,{}^{sc}H_g^a)(\bar{\xi}) \notin \mathrm{WF}_{Sc}(u)$ and for all s satisfying*

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[1]All proposition, equation, etc., numbers of the form $n.m$ refer to [2].

$0 \le s \le s_0$, $\exp(-s^{sc}H_g^a)(\bar\xi) \in U$ (i.e. the integral curve segment of ${}^{sc}H_g^a$ is completely inside U), and in addition the implication

$$\xi \in U \text{ and } \eta_a(\xi) < 0 \Rightarrow \xi \notin \mathrm{WF}_{Sc}(u) \tag{1}$$

holds, then in fact $\bar\xi \notin \mathrm{WF}_{Sc}(u)$.

(ii) *(Tangential propagation is not possible.) Suppose $(\bar y_a, 0, \bar\tau_a, \bar\mu_a, 0) \notin \Sigma_b(\lambda)$ for any b. If there exists a neighborhood $U \subset U_0$ of $\bar\xi$ in $\dot\Sigma(\lambda)$ such that the implication*

$$\xi \in U \text{ and } \eta_a(\xi) < 0 \Rightarrow \xi \notin \mathrm{WF}_{Sc}(u) \tag{2}$$

holds, then in fact $\bar\xi \notin \mathrm{WF}_{Sc}(u)$.

Proof. As already remarked, (i) was proved in [1], so we turn to (ii). As the changes are minor compared to case (ii) of Proposition 7.1, we simply point out what these are. The calculation of (7.1) there is valid, i.e.

$${}^{\mathrm{sc}}H_g^b(z_a \cdot \nu_a) = (x_a/x)(2\tau_a(z_a \cdot \nu_a) + 2|\nu_{ab}|^2) > 0 \tag{3}$$

(near $\hat\pi_b^{-1}(\bar\xi)$) since $z_a \cdot \nu_a = 0$ at $\bar\xi$, while $\nu_{ab} \neq 0$ on $\hat\pi_b^{-1}(\bar\xi)$ by assumption. Since $\hat\pi_b^{-1}(\bar\xi)$ is compact, we let, as in (7.6),

$$c_0 = \frac{1}{2}\inf\{{}^{\mathrm{sc}}H_g^b(z_a \cdot \nu_a)(\bar\xi_b) :\ \bar\xi_b \in \hat\pi_b^{-1}(\bar\xi)\} > 0. \tag{4}$$

The only change we need now is in the definition of ω – in fact, too many unnecessary conditions were arranged for ω in [2]. So we let

$$\omega = (\xi_a - \bar\xi_a)^2 + (y_a - \bar y_a)^2, \tag{5}$$

corresponding to taking $C = 0$ in [2]. Now ${}^{\mathrm{sc}}H_a^g\omega = 4\mu_a \cdot (y_a - \bar y_a)$, so

$$|{}^{\mathrm{sc}}H_a^g\omega| \le C_1'|y_a - \bar y_a| \le C_1'\omega^{1/2} \tag{6}$$

on $\dot\Sigma(\lambda)$. As ${}^{\mathrm{sc}}H_g^b|z_a|^2 \le C_1|z_a|$ still, (7.22) becomes

$${}^{\mathrm{sc}}H_g^b\phi \ge {}^{\mathrm{sc}}H_g^b(z_a \cdot \nu_a) - 2\sqrt{\beta}(C_1 + C_1') > 0 \text{ if } \beta = (c_0/4(C_1 + C_1'))^2. \tag{7}$$

With this new β, the rest of the argument in the proof of Proposition 7.1 is unchanged. $\square$

This completes the proof of Theorem 2.2 of [2].

Remark 1. In (i) of Proposition 1, both tangential and normal propagation are possible, while in (ii) only normal propagation, hence the relatively simpler statement of (ii).

Also, $\lambda \notin \Lambda_1$ was assumed in Theorem 2.2 and Proposition 7.1 of [2], but this is an unnecessary assumption. However, it becomes necessary for the statements about the resolvent limits $R(\lambda \pm i0)$ or the scattering matrices, Theorem 2.4 and Theorem 2.8, since otherwise the limits do not even exist.

References

[1] A. Vasy. Propagation of singularities in many-body scattering. *Ann. Sci. École Norm. Sup. (4)*, 34:313–402, 2001.

[2] A. Vasy. Propagation of singularities in many-body scattering in the presence of bound states. *J. Func. Anal.*, 184:177–272, 2001.