

## MATH 216A. ALGEBRAIC GEOMETRY

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**Office hours:** MWF, 4-5pm

**Prerequisites:** Math 210A & 210B, and some basic awareness about manifolds

**Textbook:** Algebraic Geometry by Hartshorne (required), Commutative Ring Theory by Matsumura (recommended)

**Course description:** Algebraic geometry was classically concerned with the geometric study of solutions to polynomial equations in several variables over  $\mathbf{C}$ . In its modern reformulation based on the concept of a scheme, the subject has acquired awesome technical power and its techniques not only permit a better arsenal with which to study classical problems over any field (not just  $\mathbf{C}$ ), but also have a vast range of applicability beyond the classical concerns: algebraic methods for studying analytic concepts, a geometric foundation that allows one to “visualize” commutative algebra and number theory, a source of important constructions and techniques in representation theory, a common framework in which one can view Galois theory and fundamental groups as “the same thing”, and so on ad infinitum.

After a couple of weeks on general considerations in Chapter I with (possibly reducible) affine algebraic sets over an algebraically closed field (to provide intuition and experience for how algebra relates to geometry and for the later use of sheaves), the course will cover the first 5 sections of Chapter II and additional topics not in the textbook. This material takes more time than the rest of Chapter II, since there are *many* definitions to be learned; 216B will get into the heart of Chapter III. It is *extremely important* to do the homework; it is the only way to really understand things. If you have time, work on as many exercises as possible in the Hartshorne textbook, not just the ones which are assigned.

In order to cover a reasonable amount of material, we assume you know commutative algebra at the level of Math 210A and Math 210B. The first 15 sections of the recommended Matsumura book (not to be confused with his book “Commutative Algebra”) constitute a more sophisticated perspective on the same material, emphasizing some technical issues (like flatness) and treating additional topics such as valuation rings, dimension theory of local noetherian rings, and completions, the first of which is relevant to this course and the other two of which are not (but are very important later on).

Beware that Hartshorne’s references to “Matsumura” are to a *different* book by the same author (“Commutative Algebra”), but the recommended Matsumura text contains all the relevant proofs too. Some commutative algebra facts may just be stated clearly in lecture with reference to “our” Matsumura text for a proof, to save time.

If you plan to learn more about schemes beyond Math 216A, please consider to *gradually read the first 15 sections of the recommended Matsumura text as the fall term progresses* (both as a review of things you’ve seen and to get a deeper understanding).

**Homework, Exams, Grades:** There will be no exams: the course grade is based entirely on homework (all counting equally towards the grade), with the lowest homework score dropped. Homework is assigned on Fridays and due *at the start of class* (on paper!) on the following Friday. Do discuss the exercises with others, but write up solutions in your own words. *Late homework will not be accepted for any reason whatsoever* (no email submission; please be responsible about printing if typed), but the lowest homework score is dropped.