

MATH 249C. ABELIAN VARIETIES

Instructor: Prof. Brian Conrad, conrad@math.stanford.edu

Office: 383CC, Sloan Hall

Office hours: WF, 3:30–5pm (usually at tea on 4th floor if not in my office).

Course assistant: None.

Prerequisites: Familiarity with the theory of schemes at the level of 216 A,B,C (e.g., valuative criteria, infinitesimal methods, representable functors, and cohomology of coherent sheaves later in the course). Knowledge of algebraic number theory will be needed in later parts of the course.

Textbooks: *Abelian varieties* by David Mumford.

Homework/exams: There will be homework posted every Friday, and undergraduates must turn it in. (The first homework is posted before the course begins, and is due on the first Friday by any undergraduates enrolled.)

Course description: We will aim to hit the highlights of the theory: complex-analytic case, general algebraic theory over an *arbitrary* field, duality and endomorphism algebras, and special results over interesting fields (finite fields, local fields, global fields). We will discuss heights and prove the Mordell–Weil theorem for an abelian variety over a global field. In view of time constraints, our discussion of the complex-analytic case will likely be more of a survey without much proof, but on the algebraic and arithmetic sides I will aim to give a complete treatment of the basics (granting familiarity with some techniques from algebraic geometry). After the course you should be able to read Tate’s paper on isogeny classes of abelian varieties over finite fields, and should do so.