Worksheet 1

Basics of counting

1. Students are taking a multiple choice quiz with 8 questions. Each question has 5 possible answers

   (a) In how many ways can a student answer all the questions on the quiz if the student answers every question?

   There are five options for every question, so 5^8

   (b) In how many ways can a student answer all the questions on the quiz if questions can be left blank?

   There are now six options for every question, so 6^8

2. How many different 2 letter initials with none of the letters repeated can people have?

   $26 \cdot 25$

3. How many bit strings of length 5 or less are there?

   63 (if you include the empty string).

4. How many four element DNA sequences

   (a) Do not contain A?

   $3^4 = 81$

   (b) Have G as the second base? $4^3 = 64$

5. How many strings of six lower case letters:

   (a) Start with cz (in that order).

   26^4

   (b) End with cz (in that order).

   26^4

   (c) Both start and end with cz (in that order).

   26^2

6. How many positive integers between 100 and 999 are divisible by 7 but not 4?

$$\lfloor \frac{999}{7} \rfloor - \lfloor \frac{100}{7} \rfloor - \lfloor \frac{999}{28} \rfloor + \lfloor \frac{100}{28} \rfloor = 96$$

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7. New Zealand license plates are either three digits followed by four letters, or four digits followed by three letters. How many possible New Zealand licenses are there?

$$10^3 \cdot 26^4 + 10^4 \cdot 26^3$$

8. Brooklyn Marble Ice Cream's Colossus of Rhodes Sundae contains five scoops of ice cream stacked in a column. If exactly 4 different flavors are used, how many different sundaes can be created?

We know that there must be an ice cream flavor with exactly two scoops. Therefore if we pretend that they're all different, we are double counting. The answer is then

$4 \cdot 5!/2 = 240$

Source: Rosen's *Discrete Mathematics and its Applications.*