Math 10B with Professor Stankova
Worksheet, Discussion #6; Tuesday, 2/12/2019
GSI name: Roy Zhao

Algorithms

Examples

1. Demonstrate bubble sort to sort the list 3, 4, 2, 1.

   Solution: On the first pass, we go $3421 \to 3241 \to 3214$, on the next pass we do $3214 \to 2314 \to 2134$ and on the third pass we do $2134 \to 1234$ and on the fourth pass, we make no changes which means the algorithm terminates.

2. Demonstrate the quick sort to sort the list $3, 6, 2, 5, 1, 4$.

   Solution: First we place the first number in the appropriate position and put the numbers smaller before it and the numbers larger after while preserving their relative order to get $362514 \to 213654$. Now we do the same on the smaller numbers and the larger to get $21 \to 12$ and $654 \to 546 \to 456$. This finally sorts the list as 123456.

3. Demonstrate the stable matching algorithm when men and women have the preferences $m_1 : w_1 > w_2, m_2 : w_1 > w_2$ and $w_1 : m_1 > m_2, w_2 : m_1 > m_2$.

   Solution: Both men will propose to woman 1 and she will choose man number 1. Then man 2 will propose to his next option which is woman 2 and she will accept him. Thus, we get the final pairing $(m_1, w_1), (m_2, w_2)$.

Problems

4. **TRUE** False The stable matching algorithm with always produce a matching that is stable.
5. True **FALSE** There is only one stable matching.
   Could be more!

6. Three women A, B, C are proposing to men E, F, G. Their preferences are as follows:

A	B	C	E	F	G
$E > G > F$	$E > G > F$	$G > E > F$	$C > A > B$	$A > B > C$	$B > C > A$

Show the stable matching algorithm with the women proposing to the men by clearly showing all rounds in a table.

Solution:

Men	Rd 1	Rd 2	Rd 3	Rd 4	Rd 5
E	A, ~~B~~	A	~~A~~, C	C	C
F					A
G	C	~~C~~, B	B	B, ~~A~~	B

7. Sort the list $2, 1, 6, 4, 5, 3$ using both bubble sort and quicksort.

Solution: Using bubble sort, we get

$$216453 \to 124536 \to 124356 \to 123456$$

Using quicksort using the last number as a pivot, we get

$$216453 \to 213645 \to (12)3(456).$$

8. Find and prove a formula for $1 + 2 + 3 + \ldots + n$

Solution: We induct. $1 + 2 + \ldots + n = \frac{n(n+1)}{2}$. For the base case, $\frac{1(1+1)}{2} = 1$. For the inductive step

$$\begin{aligned} 1 + 2 + \ldots + n + (n+1) = \frac{n(n+1)}{2} + n + 1 = \frac{n^2+n}{2} + \frac{2n+2}{2} &= \frac{n^2+3n+2}{2} \\ &= \frac{(n+1)(n+2)}{2} \end{aligned}$$

as desired.