Worksheet 7

Inductions

1. Prove using mathematical induction that for all $n \geq 1$,

$$1 + 4 + 7 + \cdots + (3n - 2) = \frac{n(3n-1)}{2}.$$

2. Prove that

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \cdots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}.$$

3. (*) Prove using mathematical induction that for all $n \geq 1$, $6^n - 1$ is divisible by 5.

4. Let $\{a_n\}_{n \geq 1}$ be a sequence defined as $a_1 = 1$ and $a_{n+1} = \sqrt{a_n + 2}$. Prove that $a_n \leq 2$ for all $n \geq 1$, by using mathematical induction.

5. Let $\{a_n\}_{n \geq 1}$ be a sequence defined as $a_1 = 1, a_2 = 5$ and $a_{n+2} = 5a_{n+1} - 6a_n$. Prove that $a_n = 3^n - 2^n$ for all $n \geq 1$, by using mathematical induction.

6. (a) Prove that $n^2 + 3n$ can be divided by 2 for every $n \geq 1$.
   (b) Prove that $n^3 - n$ can be divided by 3 for every $n \geq 1$.

7. (a) (*) Let a_n be the number of permutation of distinguishable n-balls. (Assume that we don't know $a_n = n!$ yet.) Prove that $a_1 = 1$ and $a_{n+1} = (n+1)a_n$.
   (b) By using the above recurrence relation and mathematical induction, prove that $a_n = n!$.