

True/False - No explanation needed. (For each: 1 point if correct, 0 points if not answered, -1 points if incorrect)

1. The Pareto distribution $f(x) = \frac{a-1}{x^a}$ for $x \geq 1$ fails to have a well defined μ when $a < 2$. True/False

True. For $a < 2$, we have $\int_0^\infty \frac{a-1}{x^{a-1}} dx$, so the antiderivative will be $\frac{a-1}{a-2} \frac{1}{x^{a-2}} \Big|_0^\infty$. But $a-2 < 0$, so this doesn't converge.

2. The second form of Ch.I. $P(|X - \mu| \geq r) \leq \frac{\text{Var}(X)}{r^2}$ can be obtained by algebraically manipulating the first form of Ch.I. $P(\mu - k\sigma < X < \mu + k\sigma) \geq 1 - \frac{1}{k^2}$, *without* invoking again integrals. True/False

True.

$$\begin{aligned} P(|X - \mu| \geq r) &= 1 - P(\mu - r < X < \mu + r) \\ &= P\left(\mu - \frac{r}{\sigma}\sigma < X < \mu + \frac{r}{\sigma}\sigma\right) \\ &= 1 - \left(1 - \frac{\sigma^2}{r^2}\right) = \frac{\text{Var}(X)}{r^2} \end{aligned}$$

Problems - Needs justification.

1. A basketball factory produces an average of 1000 basketballs a day with a variance of 100. Give a lower bound on the probability that on a given day, the factory produces between 950 and 1050 basketballs.

$\mu = 1000$. $\sigma = 10$. Therefore $P(\mu - 5\sigma < X < \mu + 5\sigma) \geq 1 - \frac{1}{25} = \frac{24}{25}$